ANALYSIS OF THE SUPPLY CHAIN DESIGN AND PLANNING ISSUES:

MODELS AND ALGORITHMS

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CONTENT

SUMMARY

As organizations globalize to reach new markets and achieve higher production and sourcing efficiencies in recent decades, supply chain design and planning play an increasingly important role in moving materials and products throughout the organizations' supply chains. An appropriate design and planning of supply chains for an organization can squeeze out the inefficiencies of the activities in the supply chain and an amount of savings is achieved consequently. Therefore, it is significant to carry out a deeper investigation in model development and algorithm design for supply chain design and planning to enhance the efficiencies of the activities in supply chains. It thus forms the focus of this thesis.

First of all, this thesis reviews the state of art on the supply chain design and planning. This literature review is classified into domestic supply chain design and planning, which includes supply chain network equilibrium models and competitive facility location problems, and global supply chain planning.

With respect to the domestic supply chain design and planning, the research of this thesis starts from supply chain network equilibrium (SCNE) models. An alterative formulation is provided for the SCNE models (Nagurney et al., 2002; Dong et al., 2004) which are formulated by variational inequalities (VIs) and solved by the modified projection method. It overcomes the difficulty in obtaining an appropriate step size for the projection method to ensure convergence. Subsequently, an SCNE model with production capacity constraints is developed. This is an important

extension to SCNE model since production capacities do have significant impacts on the decisions of manufacturers. A Mathematical Program with Equilibrium Constraints (MPEC) model is subsequently developed for a competitive facility location problem, applying the SCNE model with production capacity constraints to derive the equilibrium state of the market. It is a novel application of SCNE model. Moreover, it is the first time a study is done on competitive facility location for a three level supply chain.

With respect to the global supply chain planning, a chance constrained programming model is established for a multiperiod global supply chain planning with consideration of transfer pricing and demand uncertainty. This model can capture the impact of fluctuation of international characteristics such as exchange rates and demand uncertainty on decisions such as transfer pricing and the after-tax profit of a multinational company (MNC). It should be pointed out that this chance constrained programming model is for only one MNC. Hence, in the last part of this thesis, a generalized Nash game model is developed for studying the competition of several MNCs that produce substitutable products. To our best knowledge, it is the first game-theoretical model that considers transfer pricing, different gradual tax brackets of different countries and other international characteristics which do affect the decisions of global supply chains.

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CHAPTER 1 INTRODUCTION

1.1 Background

Developments in the field of production management since World War II have been limited to the improvement of activities related to production control and design in individual functional areas such as inventory management, planning and scheduling of manufacturing activities, modeling and evaluation of manufacturing systems, layout problems, group technology, system design approaches, and design and control of information flows. In those years, manufacturers mainly concentrated on the production technology revolutions. In recent decades, as organizations globalize to reach new markets and achieve higher production and sourcing efficiencies, supply chain management have played an increasingly important role in moving materials and products throughout the organizations' supply chains. Effective decisions of supply chain can give an organization benefits such as distribution savings, greater control of business, better customer service and satisfaction, and reduction in capital investment in facilities, equipment and information technology.

Nowadays, the definition of a supply chain can legitimately be broad or narrow, depends on the perspective of the "definer". In this dissertation, a supply chain is defined as an integrated process wherein a number of various business entities, such as suppliers, manufacturers, distributors, customers, work together in an effort to: (1) acquire raw materials, (2) convert these raw materials into specified final products, and (3) deliver these final products to customers (Beamon, 1998). This chain, as shown in figure 1.1, is traditionally characterized by a forward flow of materials and a backward flow of information.

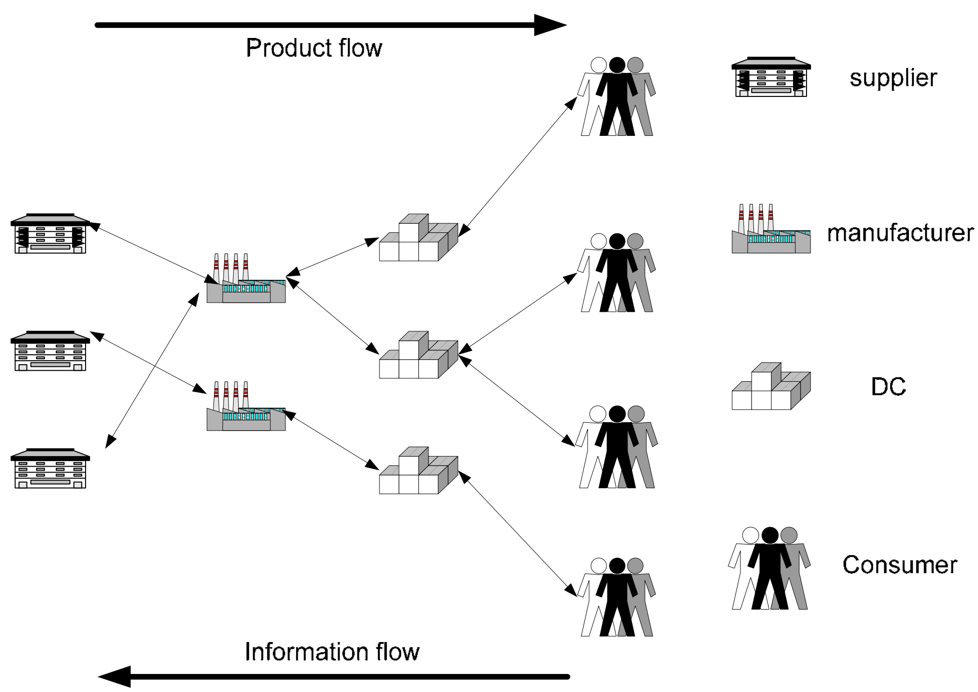


Figure 1.1 An example of supply chains

Generally, decisions of supply chain can be divided into three levels in terms of planning horizon: strategic level, tactical level and operational level (Goetschalckx et al., 2002). The strategic level usually considers time horizons of more than one year, including the determination of facility locations, production technologies and facility capacities. Normally it is denoted as supply chain design. The tactical level focuses on material flow management policies such as production levels at each plant, assembly policy, inventory levels and lot sizes. Normally it is termed as supply chain planning. The operational level, which is always denoted as supply chain execution or implementation, schedules operations to assure in-time delivery of final products to customers, coordinating the logistics network to be responsive to customer demands. This thesis only studies strategic level and tactical level decisions of supply chain, namely, supply chain design and planning. Up to date, mathematical models are widely used in supply chain decisions. For example, they are widely used in demand forecasting and data mining. Model practitioners always develop optimization models to better understand functional relations in the company and the outside world (Shapiro, 2007). An appropriate design and planning of supply chains for an organization can squeeze out the inefficiencies of the activities in the supply chain and a certain amount of savings is achieved consequently. As such, it is worth conducting research on the models and algorithms of supply chain design and planning.

1.2 Objectives

This thesis focuses on the supply chain design and planning, which are approached broadly from two perspectives, domestic supply chain and global supply chain. The former one refers to supply chain design and planning without consideration of international characteristics such as currency exchange rates, import duties and local contents, while the later one refers to supply chain planning taking those international features into account.

1.2.1 Domestic supply chain

The study on domestic supply chain in this thesis focuses on the models, algorithms and applications of supply chain network equilibrium (SCNE) models. SCNE models are originally proposed by Nagurney and her collaborators in 2002.

They have been widely used in supply chain studies such as reverse logistics (Nagurney and Toyasaki, 2005) and global supply chain planning (Nagurney et al, 2003). Therefore, it is worth exploring the alternative formulation and algorithm for the SCNE models.

The SCNE models (Nagurney, et al., 2002; Dong et al., 2004) are formulated by variational inequalities (VIs) and solved by the modified projection method. At each iteration of the modified projection method a predetermined step size is needed to implement the projection. However, a universal step size guaranteeing the convergence of the modified projection method does not exist because it relies on the unknown Lipschitz constant of the vector function entering a VI formulation. In other words, while implementing the modified projection method, it is a challenging issue to obtain a desirable step size. Therefore, Chapter 3 transforms the SCNE models to unconstrained minimization problems by using Fischer function (Fischer, 1992). Hence quasi-Newton algorithm can be applied to solve this problem. It should be pointed out that the technique proposed in Chapter 3 is not only applicable to the two cases studied in Chapter 3, but to all of the other SCNE models because all of these SCNE models were formulated by VIs defined on nonnegative orchant (e.g. Nagurney, et al., 2003 and Nagurney and Toyasaki, 2005).

In addition, a manufacturing facility, in fact, should have the production capacity constraint, i.e., a limit on the amount of the product produced during a time period, due to the limited resources. However, the SCNE model (Nagurney et al., 2002) does not take into account production capacities for manufacturers. Hence, Chapter 4 extends the SCNE model to an SCNE model with production capacity constraints.

Competitive facility location problems are to make decisions on facility locations for companies while taking into account the interactions between location decisions and market forces. Up to now only the spatial price equilibrium (SPE) (Nagurney, 1999) model or Cournot-Nash Oligopolistic equilibrium model is applied in competitive facility location problems to describe the economic equilibrium state of the market. Tobin and Friesz (1986) proposed the competitive facility location issue that is able to quantitatively take into account the market competition to some extent. They developed a generalized bilevel programming model for the competitive facility location problem, in which the lower level problem is the SPE model or Cournot-Nash Oligopolistic equilibrium model that characterizes the economic equilibrium state of the market in response to the facility location decision of an entering firm.

After a series of explorations in depth (Friesz et al., 1988 and 1989; Miller et al. 1992), Miller et al. (1996) contributed a monograph on the competitive facility location problems with SPE constraints, and pointed out that bilevel programming models and sensitivity analysis based heuristic methods can provide a solution to the competitive facility location problem. However, although the SPE model or Cournot-Nash Oligopolistic equilibrium model can quantify the supply and demand equilibrium conditions, it is incompetent on capturing economic equilibrium conditions of a supply chain comprising manufacturers, retailers and consumers with free-market competition. As such, a novel and interesting research issue regarding the competitive facility location on the decentralized supply chains has emerged. In

Chapter 4, after obtaining the SCNE model with production capacity constraints, a Mathematical Programming with Equilibrium Constraints (MPEC) model for a competitive facility location problem was developed, applying the SCNE model with production capacity constraints to derive the economic equilibrium state of a supply chain comprising manufacturers, retailers and demand markets.

1.2.2 Global supply chain

The objective of study on global supply chain in this thesis is to conduct research on some new global supply chain planning issues.

As is known, transfer pricing and the allocation of overhead of a multinational company (MNC) can shift profit of its subsidiaries located in high-tax countries to its subsidiaries located in low-tax countries. These thus would increase the after-tax profit of this MNC. Transfer price here is defined as the price that a selling department, division, or subsidiary of a company charges for a product or service supplied to a buying department, division or subsidiary of the same company (Abdallah, 1989). Although some articles conducted research on this issue (Cohen et al, 1989; Vidal and Goetschalckx, 2001 and Wilhelm et al., 2005), they ignore that currency exchange rates may fluctuate over a taxation period. This fluctuation may affect the decisions of MNCs. Moreover, the market demand considered in the three articles was assumed to be deterministic. Therefore, in Chapter 5 a chance constrained programming model was proposed for a multiperiod production- distribution planning for an MNC with consideration of transfer pricing and demand uncertainty.

In reality, MNCs that produce substitutable products may compete with each other. For instance, in the personal computer industry, three giant MNCs - Dell, Hewlett-Packard and Lenovo - are competing with each other worldwide because they assemble highly substitutable desktop computers in their plants and sell them to consumers via their distribution centers (DCs). To be more competitive, these companies have already put their plants and DCs in different countries or territories, which form a two-echelon global supply chain concerning international features such as currency exchange rates, import duties, transfer prices, tax brackets and transportation cost allocation. However, to the best of our knowledge, up to now no academic research has been conducted on the competition of the MNCs that minimize their respective after-tax profit through transfer pricing and allocating the transportation cost among their respective subsidiaries. Hence, in Chapter 6 a generalized Nash game model is proposed to analyze the competition among MNCs that produce substitutable products with consideration of transfer pricing, allocation of transportation cost and gradual tax brackets.

1.3 Outline of the Thesis

This thesis is organized as follows:

Chapter 2 gives a comprehensive literature review of the SCNE models, competitive facility location problems and global supply chain planning.

Chapter 3 transforms the VI formulation for the SCNE models into unconstrained minimization problems. Subsequently, the quasi-Newton algorithm is applied to solve them. An illustrative numerical example is presented to evaluate the convergence of quasi-Newton algorithm and the modified projection method. Furthermore, ten benchmark numerical examples are applied to compare the computational time of quasi-Newton method and the modified projection method.

Chapter 4 first proposes an SCNE model with production capacity constraints. Based on this model, it develops an MPEC model for a competitive facility location problem. GA incorporated with LQP P-C method is designed to solve this MPEC model. Finally, sensitivity analysis of the facility investment budget is studied.

Chapter 5 focuses on a multiperiod production-distribution planning for an MNC taking into consideration of transfer pricing and demand uncertainty. A chance-constrained programming model is developed to formulate this problem. Since the objective function is nondifferentiable and it is difficult to evaluate the violation of chance constraints, a heuristic that is a penalty method embedded with simulated annealing procedure is proposed to solve this model. Furthermore, a numerical example is employed to evaluate the impact of demand uncertainty and confidence levels of chance constraints on the after-tax profit, and ten randomly generated numerical examples are used to access the computational time of the heuristic.

Chapter 6 presents a generalized Nash game model to analyze the competition of MNCs that produce substitutable products by taking into account transfer pricing, allocation of transportation cost and gradual tax brackets for each MNC. Two heuristic algorithms are proposed to solve this model. The impact of change of currency exchange rates and gradual tax brackets on the equilibrium state are studied.

Furthermore, the convergence of these two heuristic algorithms is investigated by using 20 numerical examples.

Chapter 7 gives conclusions of this study, contribution of this thesis, and some possible research directions for further study.

CHAPTER 2 LITERATURE REVIEW

In this chapter, a comprehensive literature review of the researches in this thesis is presented. The review is classified into two sections: the review of domestic supply chain and the review of global supply chain. The review of domestic supply chain includes the models and algorithms of SCNE models and competitive facility location problems, while the review of global supply chain focuses on the models and algorithms for global supply chain design and planning.

2.1 Domestic Supply Chain

In this thesis, the research of domestic supply chain design and planning focuses on the models, algorithms and the application of SCNE models. With reference to the application of SCNE models, SCNE models was applied to study competitive facility location problems. Therefore, firstly, a literature review of SCNE models is presented in 2.1.1. Subsequently, a literature review of competitive facility location problems is presented in 2.1.2.

2.1.1 Supply chain network equilibrium models

The definition of SCNE was originally proposed by Nagurney and her collaborators in 2002. It describes an equilibrium state for a three-echelon supply chain comprising manufacturers, retailers and the customers. The manufacturers produce substitutable products and supply them to the retailers. In order to maximize his profit, each manufacturer makes decision on the production amount and the amount of shipment supplied to each retailer. The retailers, in turn, receive the products from the manufacturers and supply them to demand markets. In order to maximize his profit, each retailer also makes decision on the amount of shipment supplied to each demand market. The customers, finally, at each demand market will determine the amount of products bought from each retailer according to the price that they are willing to pay, the price charged by the retailers and the transaction cost. These noncooperative behaviors of manufacturers, retailers and the customers at demand markets drive the supply chain to an equilibrium state, namely, the SCNE. At equilibrium, each entity of the three-echelon supply chain cannot increase his own profit by changing his decision unilaterally. A VI formulation was developed to obtain the SCNE solution. The sufficient condition of the existence and uniqueness of the equilibrium was obtained and the modified projection method was applied to solve this SCNE model.

Subsequently, SCNE model is widely used for analyzing various supply chain issues. Nagurney et al. (2003) applied it in global supply chain by incorporating currency exchange rate into the VI formulation. Nagurney and Toyasaki (2003) obtained the SCNE solution for a supernetwork in which manufacturers not only supply products to retailers through physical links, but also supply products to demand markets directly through internet links. Also the environmental criteria were considered in this model, namely, the generated emission was incorporated into the objective function of manufacturers and retailers by assigning a negative weight. In addition, Nagurney and Toyasaki (2005) applied the idea of SCNE for a reverse supply chain management and electronic waste recycling problem in which the reverse supply chain consists of four tiers: sources, recyclers, processors and demand market.

Moreover, the idea of SCNE was also applied in studying electric power supply chain instead of traditional supply chain which always consists of such as manufacturers, retailers and demand markets (Wu et al., 2006, Nagurney et al.,2006, Nagurney et al.,2007), studying internet advertising (Zhao et al., 2008) as well as studying financial networks (Nagurney and Ke, 2006, Cruz et al., 2006).

It should be pointed out that the market demands in the above articles about SCNE are assumed to be deterministic. However, sometimes the demand cannot be predicted precisely. Therefore, it is necessary to study the SCNE with demand uncertainty. Dong et al. (2004) addressed an SCNE model with random demands. They assumed that the demand faced by each retailer is uncertain and developed a VI formulation for the SCNE model with random demands. Moreover, Dong et al. (2005) derived the SCNE solution of a four-echelon supply chain consisting of manufacturers, distributors, retailers and demand markets. This is the first SCNE model that captured both multicriteria decision-making and decision-making under uncertainty. More specifically, each manufacturer is not only focused on the profit, but also on the market share. Nonnegative weights were assigned to the market share and the objective of each manufacturer was to maximize a combination of profit and market share. The distributor was concerned with the profit, the transportation time and the service level and wanted to maximize a combination of these three objectives by assigning weights to these objectives. The retailers, in turn, wanted to maximize their respective profit while facing demand uncertainty at demand markets. Subsequently, Nagurney and Matsypura (2005) obtained the equilibrium solution of a four-echelon supply chain: manufacturers, distributors, retailers and demand markets. They considered not only the uncertainty of demand, but also the supply risk of manufacturers and distributors.

Overall, SCNE models have been being an interesting research topic nowadays. However, these SCNE models were formulated by VI formulations and solved by the modified projection method. While implementing the modified projection method, a predetermined step size is needed to guarantee the convergence of it. Up to now no efficient strategy but trial-and-error can derive such a step size. Furthermore, in some cases the required step size does not exist. In other words, a universal step size for guaranteeing the convergence of the modified projection method for solving the SCNE models is difficult to derive.

In addition, production capacities of manufacturers are necessary constraints in supply chain design and planning. They may affect the SCNE solution. However, the SCNE models have not taken into account the production capacity constraints.

2.1.2 Competitive facility location problems

Competitive facility location problems aim to make decisions on facility location for companies while taking into account the interactions between location decisions and market forces. A common assumption of it is that all of the facilities, whether newly located or already existed, are producing one homogeneous or substitutable product and compete with each other. In general, in competitive facility location problems, the decision variables include the location of facilities and the outputs of each facility. Sometimes the prices of these outputs at each facility are taken as decision variables.

Generally speaking, there are four major components for competitive facility location problems. The first component is the space, namely, whether the space available to the companies for locating a facility is discrete or continuous. Discrete spaces are always represented by the nodes of a supply chain or transportation network, while continuous spaces are always described by a space in a coordinate system whose dimension is no more than 3. The second component specifies the market rules which indicate whether the market is initially empty and all competitors enter the market simultaneously, or there already exist some competitors and an entering firm dedicates to enter the market. In Table 2.1 these two rules were termed as “simultaneously” and “sequentially”, respectively.

The third component considered in competitive facility location problems is the behaviors of customers. This term refers to how customers choose products. For instance, some customers may choose the cheapest products, some may choose the products which are nearest to them. The fourth and the last major descriptor is that of the objectives such as profits, market shares, investment ratio and service level of the decision makers. The history of competitive facility location problems dates back to the seminal paper authored by Hotelling in 1929. It sparked a good deal of activity at that time, including the papers authored by Hoover (1936), Lerner and Singer (1937) and Smithies (1941). After an ebb in the following three decades, up until the early 1970s, a resurgence of interest in competitive facility location problems appears from late 1970s to date. To summarize, a considerable body of representative articles for competitive facility location problems is presented in Table 2.1 according to the four major components presented above.

Table 2.1 Major components considered in selected competitive facility location models

Paper	Space	Market rules	Customers	Objectives
Hotelling (1929)	Continuous	Simultaneously	Price	Profit
Hakimi (1983)	Discrete	Sequentially	Distance	Market share
Revelle (1986)	Continuous	Sequentially	Distance	Market share
Hurter and Lederer (1985)	Continuous	Simultaneously	Price	Profit
Lederer and Hurter (1986)	Continuous	Simultaneously	Price	Profit
Lederer and Thisse (1990)	Discrete	Simultaneously	Price and marginal cost	Profit
Zhang (2001)	Continuous	Sequentially	Price	Profit
Garcia Perez and Pelegrin (2003)	Discrete	Simultaneously	Price and transportation cost	Profit
Fernandez et al. (2007)	Discrete	Simultaneously	Price and distance	Profit

In the models listed in Table 2.1, customers choose the products according to the factors such as prices, distances and costs. On the other hand, there is another way to describe the behaviors of the customers and the supply entities in supply chains, namely, to integrate or link an economic equilibrium model with a fixed demand facility location model to create a bilevel programming model or an MPEC model for competitive facility location problems.

Tobin and Friesz proposed a bilevel programming model in 1986 to formulate a competitive facility location problem for a firm who wants to locate its supply facilities to maximize his profit. After locating the facilities, the market, which consists of suppliers and customers, followed SPE or Cournot-Nash Oligopolistic equilibrium (Nagurney, 1999). A heuristic algorithm that is to transfer the bilevel programming model to a single level programming model by using sensitivity analysis was developed to solve this model. Subsequently, Friesz et al. (1988) developed another exact algorithm to solve the model and the existence theory for the model was studied by Friesz et al. in 1989. Finally, Miller et al. (1992) expands the competitive facility location model developed by Friesz in 1986 by introducing some transshipment nodes. It should be pointed out that these competitive facility location problems are concerned with a supply chain with only two levels: sellers and buyers. Nowadays, as companies globalize, supply chain becomes more and more complex. It does not include only sellers and buyers. Therefore, it is worth conducting research on the competitive facility location problems by linking the SCNE model (Nagurney et al., 2002) and the fixed demand location models.

2.2 Global Supply Chain

In recent years, decision makers of companies have been seeking out international manufacturing sources because of reduced cost, increased revenues and improved reliability. For instance, manufacturers set up factories in foreign countries to benefit from tariff and trade concessions, low cost direct labor, capital subsidies and reduced logistics cost. Comparing to domestic supply chain, global supply chain is more difficult to manage because many international components such as corporate income taxes (Hodder and Dincer, 1986; Arntzen et al., 1995), duties (Breitman and Lucas, 1987; Cancel and Khumawala, 1996; Lowe et al., 2002), currency exchange rates (Cohen and Lee, 1989; Haug, 1992; Nagurney et al., 2003), trade barriers (Breitman and Lucas, 1987; Munson and Rosenblatt, 1997;) and transfer prices (Cohen et al, 1989; ; Vidal and Goetschalckx, 2001; Wilhelm et al, 2005) need to be taken into account.

From modeling point of view, mixed integer programming (MIP) is the most useful approach for global supply chain design and planning. They are always solved by applying branch-and-bound algorithm or meta-heuristics such as GA. In addition, there are some other approaches which are applied in global supply chain design and planning, e.g. dynamic programming for multiperiod problems, solved by forward or backward recursion, VI formulation solved by the modified projection method and game-theoretical approach (Tombak, 1995; Dasu and de la Torre, 1997) for analyzing competition in global supply chains.

The objectives that are considered in global supply chain design and planning are also diversified. Since different tax authorities gain different corporate income tax rates, a typical objective function in global supply chain design and planning is to maximize the after-tax, even is to maximize the mean-variance of the after-tax profit while involving stochastic issue in global supply chain design and planning. In addition, lead time is another important issue in global supply chain design and planning because the shipments always move across borders for such a long distance. Hence, in some cases the objective is to minimize the weighted activity time. Besides, the other objectives in global supply chain design and planning are more or less the same as the objectives in domestic supply chain design and planning, for instance, to minimize sum of various costs.

Table 2.2 summarizes the approaches used in global supply chain design and planning, and the objectives of the models for some typical articles. It should be pointed out that for modeling approach in Table 2.2, MIP refers to mixed integer programming, Dynamic refers to dynamic programming, Game theory refers to game-theoretical model and VI refers to variational inequality.

Table 2.2 Approaches and objectives of global supply chain design and planning

Article	Modeling approach	Objective
Hodder and Dincer, 1986	MIP	Maximize mean-variance of the after-tax profit
Breitman and Lucas, 1987	MIP	Maximize profit
Haug, 1992	MIP	Minimize sum of various costs
Kougut and Kulatilaka, 1994	Dynamic	Minimize sum of various costs
Arntzen et al., 1995	MIP	Minimize the combination of weighted cost and transportation time
Tombak, 1995	Game theory	Maximize profit
Canel and Khumawala, 1996	MIP	Maximize after-tax profit
Huchzermeier and Cohen, 1996	Dynamic	Maximize after-tax profit
Dasu and de la Torre, 1997	Game theory	Maximize profit
Munson and Rosenblatt, 1997	MIP	Minimize sum of production and purchase cost
Kouvelis et al., 2001	Dynamic	Maximize profit
Nagurney et al., 2003	VI	Maximize profit
Souza et al., 2004	MIP	Maximize profit
Nagurney and Matsypura, 2005	VI	Maximize profit

From the point of view of the factors that may affect global supply chain design and planning, there are two kinds of factors, deterministic factors and stochastic factors. Deterministic factors include such as production costs, transportation costs, transportation modes, inventory costs and capacities while stochastic factors include such as market demands, currency exchange rates and market prices. Early research on the stochastic issues of global supply chain appears in Hodder and Jucker (1982 & 1985) and Hodder and Dincer (1986). They stated that the market price of the products and the currency exchange rates are uncertain and utilize mean-variance approach to measure the decision maker's risk. Since the problems in these papers are single period problem, they cannot measure the impact of the fluctuation of currency exchange rate on global supply chain design and planning. Other articles taking into account uncertain currency exchange rates in global supply chain design and planning include such as Kogut and Kulatilaka (1994) and Huchzermeier and Cohen (1996). Both of them assume that currency exchange rate follows a Wiener process and hence the currency exchange rate in each discrete time depends on the currency exchange rate in the previous period. Except for the exchange rate and price, many other random features such as uncertain demand (Sodhi, 2005) and political risk (Nagurney and Matsypura, 2005) have been explored in global supply chain design and planning.

In general, there are two savings potential while planning a global supply chain. One is the difference of cost, such as production cost, labor cost and transportation cost, in different countries or territories (e.g. Hodder and Dincer, 1986; Arntzen et al., 1995; Huchzermeier and Cohen, 1996; Kouvelis et al., 2001 and Souza et al., 2004).

These factors may help to decrease the cost much more than in domestic issues because the costs between countries, especially developing countries and developed countries, are quite different. Another saving originates from the tax savings. More specifically, since the tax rates in different countries are different, it is possible to shift the profit from the subsidiaries in high-tax countries to the subsidiaries in low-tax countries through transfer pricing and allocating overhead of an MNC (Cohen et al, 1989, Vidal and Goetschalckx, 2001). In 2005, Wilhelm and his collaborators stated that corporate tax rate of the profit is not a constant, but a step-wise function of the profit. Namely, it is more applicable to include gradual tax brackets in global supply chain planning while considering transfer pricing and allocation of transportation cost to reduce income tax.

The three articles studying transfer pricing for an MNC cannot capture the fluctuation of currency exchange rate on global supply chain planning. Moreover, they assumed that the demand at the demand market was deterministic. However, in most of the cases the demand cannot be predicted precisely. Therefore, it is worth conducting research on a multiperiod supply chain planning for an MNC with the consideration of transfer pricing and demand uncertainty.

On the other hand, so far the global supply chain planning with consideration of transfer pricing is for only one MNC. In other words, it is for a centralized supply chain. In reality, MNCs that produce substitutable products always compete with each other. In other words, the global supply chain is decentralized. To the best of our knowledge, the first result on competition for the global supply chain planning was developed by Tombak (1995). With linear demand assumption, Tombak (1995) proposed a deterministic differentiable game-theoretical model to analyze when MNCs would switch from exporting to producing at an onshore plant for the case of two MNCs. It aims to determine Nash equilibrium timing patterns with resorting to Cournot equilibrium production quantity and selling price at each period. However, Tombak (1995) disregarded the unique and important international features that definitely have vital impact on planning a global supply chain. With currency exchange rates, tariff rates and transfer prices, Dasu and Torre (1997) developed a static Nash game model to characterize the equilibrium solution of the decentralized global supply chain in the context of textile fiber producers in Latin American, in which each MNC attempted to maximize his own profit without consideration of tax issues. These two game models unfortunately ignore the income tax rates published by countries involved in the decentralized global supply chain and transportation cost allocation ratios between plants and DCs belonged to the same MNC. These two international features not only affect after-profit of an MNC but also make global supply chain planning fairly different in model development and algorithm design. Overall, it is necessary to propose a game-theoretical model for MNCs that produce substitutable products and compete with each other with the consideration of transfer pricing, allocation of transportation cost and gradual tax brackets.

CHAPTER 3 REFORMULATING SUPPLY CHAIN NETWORK EQUILIBRIUM MODELS

3.1 Introduction

In this chapter, an alternative formulation and solution algorithm for the SCNE model (Nagurney et al., 2002) and the SCNE model with demand uncertainty (Dong et al., 2004) are provided. Moreover, 11 numerical examples are used to evaluate this solution algorithm suggested in this chapter.

3.2 Supply Chain Network Equilibrium Models

In this section, the SCNE model (Nagurney et al., 2002 & Dong et al., 2004) are introduced. Let us consider a three-tier decentralized supply chain network comprising manufacturers, retailers and consumers for a homogenous or substitutable product, depicted by Figure 3.1 (Nagurney et al., 2002). In the network, nodes in the top tier represent manufacturer producing the product, and nodes in the middle tier denote retailers who purchase a certain amount of the product from the manufacturers and then sell them to consumers located at the demand markets shown in the bottom tier. Directed links indicate transportation and/or transaction relations of the product among the decision-makers in the supply chain. Assume that there are m manufacturers, n retailers and o demand markets in the supply chain. Without loss of generality, a typical manufacturer, retailer and demand market are denoted by notations i, j, k, respectively.

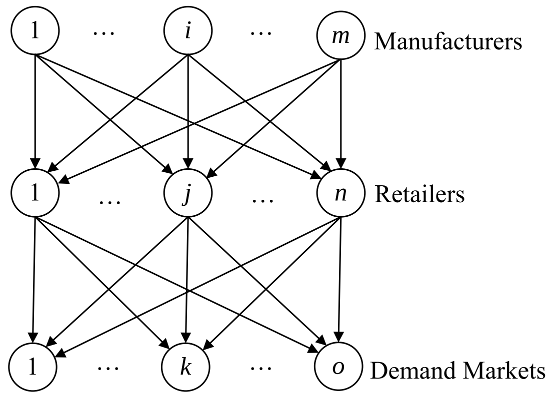


Figure 3.1 Network structure of the supply chain with deterministic demands

The aim of manufacturer i is to maximize his profit by determining his production output denoted by q_i, shipment of the product shipped or transacted to retailer j denoted by q_{ij}. Cost for producing the product of manufacturer i can be in general described by function $f_i(q)$, where $q = (q_1, \cdots, q_m)$ is the row vector of production outputs of all manufacturers in the supply chain. The transaction cost of the product between manufacturer i and retailer j is characterized by function $c_{ij}(q_{ij})$. It is assumed that the quantity of the product produced by manufacturer i is equal to the sum of the quantities shipped from the manufacturer to all retailers, namely:

$$q_i = \sum_{j=1}^{n} q_{ij}, i = 1, \cdots, m \tag{3.1}$$

For the notational convenience, let Q^1 be the mn-dimensional row vector of all product shipments between manufacturers and retailers, i.e., $Q^1 = (\cdots, q_{ij}, \cdots)$, $i = 1, \cdots, m$ and $j = 1, \cdots, n$. As such, production cost function $f_i(q)$ for manufacturer i can be alternatively regarded as a function of vector Q^1, i.e. $f_i(Q^1)$, according to eqn. (3.1).

It is assumed that the manufacturers as the profit-maximizers in the supply chain compete in a noncooperative fashion (Nash game) and that supply price of the product is identified according to the marginal-cost pricing principle. Furthermore, Assumed that the production cost function and the transaction cost function for each manufacturer are continuously differentiable and convex. The product quantities and shipments of all manufacturers in the equilibrium state following the Nash game-theoretical principle can be thus determined by solving the VI (Nagurney et al., 2002): Find a vector $Q^{1*} \in \Re_{+}^{mn}$ satisfying the inequality:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial f_i\left(Q^{1*}\right)}{\partial q_{ij}}+\frac{\partial c_{ij}\left(q_{ij}^{*}\right)}{\partial q_{ij}}-\rho_{1ij}^{*}\right]\times\left[q_{ij}-q_{ij}^{*}\right]\geq 0,\quad \forall Q^{1}\in\Re_{+}^{mn} \tag{3.2}$$

where $\Re_{+}^{mn}$ is the nonnegative orthant in the *mn*-dimensional real space $\Re^{mn}$.

3.2.1 Deterministic demand case

Consumers grouped into different demand markets in the supply chain consume the product according to their own consumption behaviors. With regard to demand market *k*, the consumers' consumption behavior for the product is assumed to be governed by deterministic demand function $d_k\left(\rho_3\right)$, where the *o*-dimensional row vector $\rho_3=\left(\rho_{31},\cdots,\rho_{3k},\cdots,\rho_{3o}\right)$ in which ρ_{3k} denotes unit price of the product that consumers in demand market k ($k=1,\cdots,o$) are willing to pay. Under the supply chain network structure shown in Figure 3.1, consumers purchase the product from retailers. Let q_{jk} be the quantity of the product bought from retailer j by consumers in demand market k, and let Q^2 be the *no*-dimensional row vector of all product flows between retailers and demand markets, i.e., $Q^2=\left(\cdots,q_{jk},\cdots\right)$, $j=1,\cdots,n$ and

$k=1,\cdots,o$. When the consumers make their consumption decisions on the product, the transaction cost to obtain the product from a retailer should be also considered. Let function $c_{jk}\left(Q^2\right)$ denote unit transaction cost of the product from retailer *j* to consumers in the demand market *k*. The spatial price equilibrium conditions for consumers located at all demand markets in the supply chain, thus, can be governed by the following VI (Nagurney et al., 2002):

Find a vector $\left(Q^{2*},\rho_3\right)\in\Re_+^{no+o}$ such that

$$\begin{aligned}&\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\rho_{2j}^{*}+c_{jk}\left(Q^{2*}\right)-\rho_{3k}^{*}\right]\times\left[q_{jk}-q_{jk}^{*}\right]+\\&\sum_{k=1}^{o}\left[\sum_{j=1}^{n}q_{jk}^{*}-d_k\left(\rho_3^{*}\right)\right]\times\left[\rho_{3k}-\rho_{3k}^{*}\right]\geq 0,\forall\left(Q^2,\rho_3\right)\in\Re_+^{no+o}\end{aligned}\tag{3.3}$$

where $\Re_+^{no+o}$ is the nonnegative orthant in the (no+o)-dimensional real space $\Re^{no+o}$, and ρ_{2j}^{*} is the price charged for the product by retailer *j*.

Retailer *j* has to simultaneously face with the manufacturers and the consumers in the process of transacting the product. He obtains the product from the manufacturers for his retail outlets from which the consumers will purchase the product. Nevertheless, the quantity of the product sold by retailer *j* does not exceed the total products obtained from all of the manufacturers, namely:

$$\sum_{k=1}^{o}q_{jk}\leq\sum_{i=1}^{m}q_{ij},\ j=1,\cdots,n\tag{3.4}$$

Various costs involved in handling the product for the retailer are called the handling cost described as function $c_j\left(Q^1\right)$. Retailer *j* aims to maximize its profit, which can be modeled by the optimization problem:

$$\text{maximize }\rho_{2j}^{*}\sum_{k=1}^{o}q_{jk}-c_j\left(Q^1\right)-\sum_{i=1}^{m}\rho_{1ij}^{*}q_{ij}\tag{3.5}$$

subject to constraint (3.4).

Assume that all retailers compete in a noncooperative manner in the retailing market of the product, and that the handling cost function for each retailer is continuously differentiable and convex. The Nash equilibrium solution for the retailers is thus equivalent to solving the following VI (Nagurney et al., 2002):

Find a vector $\left(Q^{1*}, Q^{2*}, \gamma^*\right) \in \Re_+^{mn+no+n}$ such that

$$\begin{aligned}&\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial c_j\left(Q^{1*}\right)}{\partial q_{ij}} + \rho_{1ij}^* - \gamma_j^*\right] \times \left[q_{ij} - q_{ij}^*\right] + \sum_{j=1}^{n}\sum_{k=1}^{o}\left[-\rho_{2j}^* + \gamma_j^*\right] \times \left[q_{jk} - q_{jk}^*\right] \\ &+ \sum_{j=1}^{n}\left[\sum_{i=1}^{m} q_{ij}^* - \sum_{k=1}^{o} q_{jk}^*\right] \times \left[\gamma_j - \gamma_j^*\right] \geq 0, \quad \forall \left(Q^1, Q^2, \gamma\right) \in \Re_+^{mn+no+n}\end{aligned} \tag{3.6}$$

where $\Re_+^{mn+no+n}$ is the nonnegative orthant in (mn+no+n)-dimensional real space $\Re^{mn+no+n}$, and n-dimensional row vector $\gamma = \left(\gamma_1, \cdots, \gamma_j, \cdots \gamma_n\right)$ in which γ_j is Lagrangian multiplier with respect to constraint (3.4) in the optimization problem (3.5).

The supply chain network involves three kinds of decision-makers: manufacturers, retailers and consumers, and they are interacted and highly correlated in the supply chain of the product, respectively. Nagurney et al. (2002) proposed a novel equilibrium concept from the point of view of entire supply chain network. The SCNE model with deterministic demands means that the production flows between the distinct tiers of the decision-makers coincide and the product flows and prices satisfy the sum of optimality conditions (3.2), (3.3) and (3.6). They further demonstrated that the SCNE model can be formulated by the following VI formulation:

Determine a vector $\left(Q^{1*}, Q^{2*}, \gamma^*, \rho_3^*\right) \in \Re_+^{mn+no+n+o}$ such that

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left[\frac{\partial f_i\left(Q^{1*}\right)}{\partial q_{ij}}+\frac{\partial c_{ij}\left(q_{ij}^*\right)}{\partial q_{ij}}+\frac{\partial c_j\left(Q^{1*}\right)}{\partial q_{ij}}-\gamma_j^*\right]\times\left[q_{ij}-q_{ij}^*\right]$$
$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[c_{jk}\left(Q^{2*}\right)+\gamma_j^*-\rho_{3k}^*\right]\times\left[q_{jk}-q_{jk}^*\right]+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}q_{ij}^*-\sum_{k=1}^{o}q_{jk}^*\right]\times\left[\gamma_j-\gamma_j^*\right] \tag{3.7}$$
$$+\sum_{k=1}^{o}\left[\sum_{j=1}^{n}q_{jk}^*-d_k\left(\rho_3^*\right)\right]\times\left[\rho_{3k}-\rho_{3k}^*\right]\geq 0,\quad \forall\left(Q^1,Q^2,\gamma,\rho_3\right)\in\Re_+^{mn+no+n+o}$$

where $\Re_+^{mn+no+n+o}$ is the nonnegative orthant in the ($mn+no+n+o$)-dimensional real space $\Re^{mn+no+n+o}$.

Having obtained the solution for the VI (3.7), the relevant equilibrium prices for the product can be identified by the formulae below:

$$\rho_{1ij}^*=\frac{\partial f\left(Q^{1*}\right)}{\partial q_{ij}}+\frac{\partial c_{ij}\left(q_{ij}^*\right)}{q_{ij}},\ \text{if } q_{ij}^*>0 \tag{3.8}$$

$$\rho_{2j}^*=\gamma_j^*,\ \text{if } \sum_{k=1}^{o}q_{jk}^*>0 \tag{3.9}$$

3.2.2 Random demand case

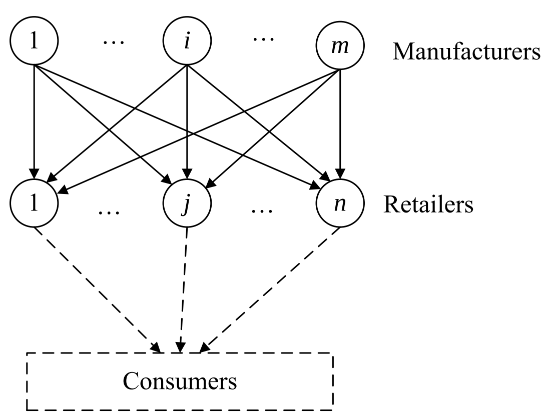


Figure 3.2 Network structure of the supply chain with random demands

Compared to the assumption of deterministic demand function utilized in the preceding SCNE model, it is more reasonable to assume that the demand for the product at a retailer outlet is a random variable. Moreover, it is not necessary to differentiate the consumers across the demand markets shown in Figure 3.1. In other words, the supply chain network with random demands explicitly considers two tiers of decision-makers: manufacturers and retailers, which are intuitively illustrated in Figure 3.2. Let $\hat{d}_j\left(\rho_{2j}\right)$ be the demand for the product with the price ρ_{2j} at retailer *j*, where $\hat{d}_j\left(\rho_{2j}\right)$ is a random variable with probability density function $\xi_j\left(x,\rho_{2j}\right)$. Therefore, the stochastic economic equilibrium conditions for all consumers in the market can be expressed by the following VI (Dong et al., 2004):

Find a vector $\rho_2^* \in \Re_+^n$ such that

$$\sum_{j=1}^{n}\left(\sum_{i=1}^{m} q_{ij}^* - d_j\left(\rho_{2j}^*\right)\right)\times\left(\rho_{2j}-\rho_{2j}^*\right)\geq 0,\quad \forall \rho_2 \in \Re_+^n \tag{3.10}$$

where $\rho_2=\left(\rho_{21},\cdots\rho_{2j},\cdots,\rho_{2n}\right)$ is the *n*-dimensional row vector of prices charged for the product by all the retailers, and $d_j\left(\rho_{2j}\right)$ is the mean value of the random variable $\hat{d}_j\left(\rho_{2j}\right)$, namely:

$$d_j\left(\rho_{2j}\right)=E\left[\hat{d}_j\left(\rho_{2j}\right)\right],\ j=1,\cdots,n \tag{3.11}$$

In the case of considering the random demands, one of two cases about excess supply and excess demand may happen with a certain probability. Let λ_j^+ be the unit penalty of having excess supply at retailer *j* and λ_j^- be the unit penalty of having excess demand. It is assumed that each retailer is a profit-maximizer and that all retailers in the supply chain compete in a noncooperative manner. The Nash equilibrium conditions for the retailers thus can be expressed by the VI (Dong et al., 2004):

Find a $Q^{1*} \in \Re_+^{mn}$ such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\lambda_j^+ P_j\left(\sum_{i=1}^{m} q_{ij}^*, \rho_{2j}\right) - \left(\lambda_j^- + \rho_{2j}\right)\left(1 - P_j\left(\sum_{i=1}^{m} q_{ij}^*, \rho_{2j}\right)\right) + \frac{\partial c_j\left(Q^{1*}\right)}{\partial q_{ij}} + \rho_{1ij}\right] \tag{3.12}$$
$$\times\left[q_{ij} - q_{ij}^*\right] \geq 0, \quad \forall Q^1 \in \Re_+^{mn}$$

where $P_j\left(\sum_{i=1}^{m} q_{ij}, \rho_{2j}\right)$ is the probability that the demand for the product at retailer j is not greater than the supply $\sum_{i=1}^{m} q_{ij}$, namely:

$$P_j\left(\sum_{i=1}^{m} q_{ij}, \rho_{2j}\right) = \Pr\left(\hat{d}_j\left(\rho_{2j}\right) \leq \sum_{i=1}^{m} q_{ij}\right) = \int_0^{\sum_{i=1}^{m} q_{ij}} \xi_j\left(x, \rho_{2j}\right) dx \tag{3.13}$$

Dong et al. (2004) defined the SCNE conditions with random demands, which is an extension of the preceding the SCNE models with the deterministic demands. They pointed out that the equilibrium state of the supply chain with random demands is one where the product flows between the two tiers of the manufacturers and the retailers coincide and the product shipments and prices satisfy the sum of the optimality conditions (3.2), (3.10) and (3.12). The relevant SCNE model can be formulated as the VI:

Determine a vector $\left(Q^{1*}, \rho_2^*\right) \in \Re_+^{mn+n}$ such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial f_i\left(Q^{1*}\right)}{\partial q_{ij}} + \frac{\partial c_{ij}\left(q_{ij}^*\right)}{q_{ij}} + \frac{\partial c_j\left(Q^{1*}\right)}{\partial q_{ij}} + \lambda_+^+ P_j\left(\sum_{i=1}^{m} q_{ij}^*, \rho_{2j}^*\right)\right.$$
$$\left. - \left(\lambda_j^- + \rho_{2j}^*\right)\left(1 - P_j\left(\sum_{i=1}^{m} q_{ij}^*, \rho_{2j}^*\right)\right)\right] \times \left[q_{ij} - q_{ij}^*\right] + \tag{3.14}$$
$$\sum_{j=1}^{n}\left[\sum_{i=1}^{m} q_{ij}^* - d_j\left(\rho_{2j}^*\right)\right] \times \left[\rho_{2j} - \rho_{2j}^*\right] \geq 0, \quad \forall\left(Q^1, \rho_2\right) \in \Re_+^{mn+n}$$

where $\Re_+^{mn+n}$ is the nonnegative orthant in (mn+n)-dimensional real space $\Re^{mn+n}$.

After getting the solution for eqn. (3.14), the equilibrium price of the product supplied to a retailer by a manufacturer can be calculated by using eqn. (3.8).

Investigating the existence and uniqueness of the solutions for VIs (3.7) and (3.14) is not the purpose of this study, but interested readers can refer to Nagurney et al. (2002) and Dong et al. (2004).

3.3 Unconstrained Minimization Formulations

According to eqns. (3.7) and (3.14), it can be easily concluded that the SCNE model with either deterministic demands or random demands can be formulated as a VI defined on a nonnegative orthant. Proposition 1.4 in Nagurney (1999) indicates that this kind of VI can be equivalently transformed into a nonlinear complementary problem (NCP). Hence, the corresponding NCP formulation for the SCNE model with deterministic demands is of the form:

Find a row vector $\tilde{X}^* \geq 0$ such that

$$\tilde{F}\left(\tilde{X}^*\right) \geq 0 \text{ and } \tilde{F}\left(\tilde{X}^*\right)\tilde{X}^{*T} = 0 \tag{3.15}$$

where row vector $\tilde{X} = \left(Q^1, Q^2, \gamma, \rho_3\right) \in \Re_+^{mn+no+n+o}$, and the row vector function $\tilde{F}\left(\tilde{X}\right)$:

$$\tilde{F}\left(\tilde{X}\right) = \left(\tilde{F}^1\left(\tilde{X}\right), \tilde{F}^2\left(\tilde{X}\right), \tilde{F}^3\left(\tilde{X}\right), \tilde{F}^4\left(\tilde{X}\right)\right) : \Re_+^{mn+no+n+o} \mapsto \Re^{mn+no+n+o} \tag{3.16}$$

where vector functions: $\tilde{F}^1\left(\tilde{X}\right) = \left(\cdots, \tilde{F}_{ij}^1\left(\tilde{X}\right), \cdots\right) \in \Re^{mn}$, $\tilde{F}^2\left(\tilde{X}\right) = \left(\cdots, \tilde{F}_{jk}^2\left(\tilde{X}\right), \cdots\right) \in \Re^{no}$, $\tilde{F}^3\left(\tilde{X}\right) = \left(\cdots, \tilde{F}_j^3\left(\tilde{X}\right), \cdots\right) \in \Re^n$ and $\tilde{F}^4\left(\tilde{X}\right) = \left(\cdots, \tilde{F}_k^4\left(\tilde{X}\right), \cdots\right) \in \Re^o$. The individual entity in these four vector functions above is defined as follows.

$$\tilde{F}_{ij}^1\left(\tilde{X}\right) = \frac{\partial f_i\left(Q^1\right)}{\partial q_{ij}} + \frac{\partial c_{ij}\left(q_{ij}\right)}{\partial q_{ij}} + \frac{\partial c_j\left(Q^1\right)}{\partial q_{ij}} - \gamma_j, \quad i = 1, \cdots, m; j = 1, \cdots, n \tag{3.17}$$

$$\tilde{F}_{jk}^2\left(\tilde{X}\right) = c_{jk}\left(Q^2\right) + \gamma_j - \rho_{3k}, \quad j = 1, \cdots, n; k = 1, \cdots, o \tag{3.18}$$

$$\tilde{F}_j^3\left(\tilde{X}\right)=\sum_{i=1}^{m}q_{ij}-\sum_{k=1}^{o}q_{jk},\quad j=1,\cdots,n \tag{3.19}$$

$$\tilde{F}_k^4\left(\tilde{X}\right)=\sum_{j=1}^{n}q_{jk}-d_k\left(\rho_3\right),\quad k=1,\cdots,o \tag{3.20}$$

With regard to VI (3.14) for the SCNE model with random demands, its NCP formulation takes the form:

Find a row vector $\hat{X}^*\geq 0$ such that

$$\hat{F}\left(\hat{X}^*\right)\geq 0 \text{ and } \hat{F}\left(\hat{X}^*\right)\hat{X}^{*T}=0 \tag{3.21}$$

where row vector $\hat{X}=\left(Q^1,\rho_2\right)\in\Re_+^{mn+n}$, and row vector function $\hat{F}\left(\hat{X}\right)$ is defined below.

$$\hat{F}\left(\hat{X}\right)=\left(\hat{F}^1\left(\hat{X}\right),\hat{F}^2\left(\hat{X}\right)\right):\Re_+^{mn+n}\mapsto\Re^{mn+n} \tag{3.22}$$

where $\hat{F}^1\left(\hat{X}\right)=\left(\cdots,\hat{F}_{ij}^1\left(\hat{X}\right),\cdots\right)\in\Re^{mn}$ with elements $\hat{F}_{ij}^1\left(\hat{X}\right)$, $i=1,\cdots,m$ and $j=1,\cdots,n$:

$$\begin{aligned}\hat{F}_{ij}^1\left(\hat{X}\right)=&\frac{\partial f_i\left(Q^1\right)}{\partial q_{ij}}+\frac{\partial c_{ij}\left(q_{ij}\right)}{q_{ij}}+\frac{\partial c_j\left(Q^1\right)}{\partial q_{ij}}+\lambda_+^+P_j\left(\sum_{i=1}^{m}q_{ij},\rho_{2j}\right)\\&-\left(\lambda_j^-+\rho_{2j}\right)\left(1-P_j\left(\sum_{i=1}^{m}q_{ij},\rho_{2j}\right)\right)\end{aligned} \tag{3.23}$$

and $\hat{F}^2\left(\hat{X}\right)=\left(\cdots,\hat{F}_j^2\left(\hat{X}\right),\cdots\right)$ with the elements $F_j^2\left(\hat{X}\right)$, $j=1,\cdots,n$:

$$\hat{F}_j^2\left(\hat{X}\right)=\sum_{i=1}^{m}q_{ij}-d_j\left(\rho_{2j}\right) \tag{3.24}$$

The merit function approach transforming an NCP into an unconstrained minimization problem is one of the efficient methods for solving NCPs (Kanzow et al., 1997). Notice that a function which can constitute an equivalent minimization problem for an NCP is called a merit function. It is very surprising that the following simple function with two variables, introduced by Fischer (1992), plays a vital role in constructing a merit function.

$$\phi(a,b) = \left[\sqrt{a^2+b^2} - (a+b)\right]^2 : \Re^2 \mapsto \Re_+ \tag{3.25}$$

It is easy to verify that this function is continuously differentiable and it has a favorable property:

$$\phi(a,b) = 0 \text{ if and only if } a \geq 0, b \geq 0, a \times b = 0 \tag{3.26}$$

With regard to NCP formulation (3.15) for the SCNE model with deterministic demands, the nonnegative real function can be constructed as follows:

$$\begin{aligned}\Psi_1\left(\tilde{X}\right) = \sum_{i=1}^{m}\sum_{j=1}^{n}\phi\left(q_{ij}, \tilde{F}_{ij}^1\left(\tilde{X}\right)\right) + \sum_{j=1}^{n}\sum_{k=1}^{o}\phi\left(q_{jk}, \tilde{F}_{jk}^2\left(\tilde{X}\right)\right) + \\ \sum_{j=1}^{n}\phi\left(\gamma_j, \tilde{F}_j^3\left(\tilde{X}\right)\right) + \sum_{k=1}^{o}\phi\left(\rho_{3k}, \tilde{F}_k^4\left(\tilde{X}\right)\right)\end{aligned} \tag{3.27}$$

The following proposition is directly derived from the eqn. (3.26).

Proposition 3.1. $\Psi_1\left(\tilde{X}^*\right) = 0$ *if and only if* $\tilde{X}^*$ *is the solution of NCP formulation (3.15).*

Proposition 3.1 actually shows that function $\Psi_1\left(\tilde{X}\right)$ is a merit function of NCP formulation (3.15). As $\Psi_1\left(\tilde{X}\right) \geq 0$, thus determining a solution for NCP formulation (3.15) is equivalent to finding a global minimum for the unconstrained minimization problem:

$$\underset{\tilde{X} \in \Re^{mn+no+n+o}}{\text{minimize}} \; \Psi_1\left(\tilde{X}\right) \tag{3.28}$$

In other words, the SCNE model with deterministic demands indeed has an unconstrained minimization formulation defined in eqn. (3.28). Using the same arguments, the SCNE model with random demands is also of the unconstrained minimization formulation:

$$\underset{\hat{X}\in\Re^{mn+n}}{\text{minimize}}\ \Psi_2\left(\hat{X}\right) \tag{3.29}$$

where the merit function $\Psi_2\left(X\right)$ is defined as follows.

$$\Psi_2\left(\hat{X}\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}\phi\left(q_{ij},\hat{F}_{ij}^{1}\left(\hat{X}\right)\right)+\sum_{j=1}^{n}\phi\left(\rho_{2j},\hat{F}_{j}^{2}\left(\hat{X}\right)\right) \tag{3.30}$$

Proposition 3.2. *The conditions in Theorem 2 of Nagurney et al. (2002) can also make sure that the unconstrained minimization problem (3.28) at least admits one solution.*

Proof. The conditions in Theorem 2 of Nagurney et al. (2002) illustrate that there exists a nonnegative vector $\tilde{B}\in\Re^{mn+no+n+o}$ such that the following VI formulation has a solution, denoted by $\tilde{X}^{\tilde{B}}$, satisfying the condition: $0\le\tilde{X}^{\tilde{B}}<\tilde{B}$.

Find a $0\le\tilde{X}^{*}\le\tilde{B}$ such that

$$\tilde{F}\left(\tilde{X}^{*}\right)\left(\tilde{X}-\tilde{X}^{*}\right)^{T}\ge 0,\ \forall 0\le\tilde{X}\le\tilde{B} \tag{3.31}$$

Since $0\le\tilde{X}^{\tilde{B}}<\tilde{B}$, there exists a sufficient small but positive number δ_1 and a row vector $\tilde{Y}=\left(\delta_2,\cdots,\delta_2\right)\in\Re^{mn+no+n+o}$ with element $\delta_2>0$ such that

$$0\le\left(1+\delta_1\right)\tilde{X}^{\tilde{B}}\le\tilde{B} \tag{3.32}$$

$$0\le\tilde{X}^{\tilde{B}}+\tilde{Y}\le\tilde{B} \tag{3.33}$$

Let us take three vectors: $\tilde{X}_1=\left(1+\delta_1\right)\tilde{X}^{\tilde{B}}$, $\tilde{X}_2=0.5\tilde{X}^{\tilde{B}}$ and $\tilde{X}_3=\tilde{X}^{\tilde{B}}+\tilde{Y}$, it follows that $0\le\tilde{X}_1,\tilde{X}_2,\tilde{X}_3\le\tilde{B}$. Substituting these three vectors into eqn. (3.31) in turn and then rearranging the induced inequalities, eqn. (3.34) can be derived as

$$\tilde{F}\left(\tilde{X}^{\tilde{B}}\right)\left(\tilde{X}^{\tilde{B}}\right)^{T}=0 \tag{3.34}$$

$$\tilde{F}\left(\tilde{X}^{\tilde{B}}\right)\ge 0 \tag{3.35}$$

Thus, it can be seen that $\tilde{X}^{\tilde{B}}$ fulfills $\Psi_1\left(\tilde{X}^{\tilde{B}}\right)=0$ according to eqn. (3.26). In other words, $\tilde{X}^{\tilde{B}}$ is a global minimum for the unconstrained minimization problem

(3.28).

Similarly, it can be demonstrated that the conditions in Theorem 3 of Dong et al. (2004) can also guarantee the existence of a global minimum for the unconstrained minimization formulation (3.29). It should be pointed out that seeking a global minimum for an unconstrained minimization problem is not easy because the minimization problem can have stationary points which are not global solutions. Fortunately, Theorem 3.5 of Geiger and Kanzow (1996) can yield:

Proposition 3.3. *If function* $\tilde{F}\left(\tilde{X}\right)$ *and* $\hat{F}\left(\hat{X}\right)$ *are monotone and continuously differentiable, then any stationary point of the unconstrained minimization problems (3.28) and (3.29) are their respective global minimums.*

Proposition 3.3 provides the confidence to implement an efficient algorithm for the unconstrained minimization problems for solving the SCNE models.

3.4 Quasi-Newton Algorithm vs. the Modified Projection Method

Quasi-Newton algorithm is one of the most efficient methods for solving the unconstrained minimization problems. It mainly consists of two components: updating an approximation of the inverse matrix of Hessian matrix of the objective function and performing a line search. More precisely, the genetic iterative schemes of quasi-Newton algorithm for the unconstrained minimization problems (3.28) and (3.29) can be presented as follows.

$$\tilde{X}^{(N+1)} = \tilde{X}^{(N)} - \tilde{\alpha}_N \tilde{D}_N \nabla \Psi_1\left(\tilde{X}^{(N)}\right) \tag{3.36}$$

$$\hat{X}^{(N+1)} = \hat{X}^{(N)} - \hat{\alpha}_N \hat{D}_N \nabla \Psi_2\left(\hat{X}^{(N+1)}\right) \tag{3.37}$$

where N is the number of iterations; $\tilde{D}_N$ and $\hat{D}_N$ are the approximations of inverse matrices of Hessian matrices, respectively; $\tilde{\alpha}_N$ and $\hat{\alpha}_N$ are the step sizes.

In reality, there are a few successful schemes such as the well-known BFGS method that can update matrices $\tilde{D}_N$ and $\hat{D}_N$ efficiently. The step sizes in eqns. (3.36) and (3.37) can be estimated by implementing a line search procedure. Here, the detailed description and the numerical examples of quasi-Newton method are not presented. Interesting reader can refer to Bazaraa et al. (1993) for details.

A projection method for solving a VI is akin to a gradient project method for solving a nonlinear programming problem to some extent. It aims at generating the iterative points closer and closer to a solution for a VI. Nagurney et al. (2002) and Dong et al. (2004) adopted the modified projection method to solve their respective VI formulations for the SCNE models. The iterative schemes of the modified projection method for VIs (3.7) and (3.14) can be stated as follows.

$$\tilde{X}^{(N+1)} = P_{\Re_+^{mn+no+n+o}}\left[\tilde{X}^{(N)} - \tilde{\alpha}\tilde{F}\left(P_{\Re_+^{mn+no+n+o}}\left(\tilde{X}^{(N)} - \tilde{\alpha}\tilde{F}\left(\tilde{X}^{(N)}\right)\right)\right)\right] \tag{3.38}$$

$$\hat{X}^{(N+1)} = P_{\Re_+^{mn+n}}\left[\hat{X}^{(N)} - \hat{\alpha}\hat{F}\left(P_{\Re_+^{mn+n}}\left(\hat{X}^{(N)} - \hat{\alpha}\hat{F}\left(\hat{X}^{(N)}\right)\right)\right)\right] \tag{3.39}$$

where $\tilde{\alpha}$ and $\hat{\alpha}$ are two predetermined parameters called step sizes, and $P_\Omega\left[Y\right]$ is the orthogonal projection of vector Y on the set Ω with respect to Euclidean norm, namely:

$$P_\Omega\left(Y\right) = \arg\min_{X\in\Omega}\left\|X - Y\right\| \tag{3.40}$$

where $\left\|X - Y\right\|$ denotes the Euclidean norm of vector $X - Y$.

In the case of $\Omega = \Re_+^{mn+no+n+o}$ or $\Omega = \Re_+^{mn+n}$, the projection operation (3.40) can be implemented easily. However, a universal step size guaranteeing the convergence of the modified projection method does not exist since it relies on the unknown Lipschitz constant of the vector function entering a VI. In other words, when implementing the modified projection method for a SCNE model, it is a challenging issue to obtain a desirable step size. Conversely, the quasi-Newton algorithm does not have such a limitation as a line search procedure is able to figure out the step size. Although performing a line search will use additional computational time, the super-linearly convergence property of the quasi-Newton algorithm may remedy it.

3.5 Numerical Examples

Up to now, this chapter has derived two unconstrained minimization formulations (3.28) and (3.29) for the SCNE models. Compared to the modified projection method, it has qualitatively shown that the quasi-Newton algorithm is more suitable to solve the SCNE models based the unconstrained minimization formulations. To evaluate the performances of these two solution methods, benchmark examples are essential. To do so, eleven examples for the SCNE models will be employed. The first benchmark example that will be constructed is a variation of Example 1 in Nagurney et al. (2002). The other ten examples those will be utilized are the same as that given by Nagurney et al. (2002) and Dong et al. (2004).

To compare the two solution methods fairly, the same initial solutions and the same stopping criterion in them for these eleven examples are adopted, which are shown as follows.

Five benchmark examples with deterministic demands:

$$\text{Initial solution: } \tilde{X}^{(0)} = (10, 10, \cdots, 10) \in \Re^{mn+no+n+o} \tag{3.41}$$

$$\text{Stopping criterion: } \left\| \tilde{X}^{(N+1)} - \tilde{X}^{(N)} \right\| \le 0.0001 \tag{3.42}$$

Six benchmark examples with random demands:

$$\text{Initial solution: } \hat{X}^{(0)} = (10, 10, \cdots, 10) \in \Re^{mn+n} \tag{3.43}$$

$$\text{Stopping criterion: } \left\| \hat{X}^{(N+1)} - \hat{X}^{(N)} \right\| \le 0.0001 \tag{3.44}$$

The modified projection method is programmed using Matlab version 6.0, and the quasi-Newton algorithm in the optimization tool box of the Matlab is invoked directly. These two solution methods are run on a personal computer with the CPU of Intel Pentium IV 1.6GMHZ and RAM of 256M.

3.5.1 A modified example

Let us consult Example 1 for the SCNE model with deterministic demands, given by Nagurney et al. (2002). Based on this example, a new numerical example is constructed as follows. It keeps all data in the original example except for the production cost functions, which now take the new expressions:

$$f_1(q) = q_1^3 + q_1 q_2 \tag{3.45}$$

$$f_2(q) = q_2^3 + q_1 q_2 \tag{3.46}$$

Applying the quasi-Newton algorithm for the relevant constrained minimization model (3.28) with respect to the modified example yields the solution: $\tilde{X}^* = \left(Q^{1*}, Q^{2*}, \gamma^*, \rho_3^*\right)$ with individual components: $Q^{1*} = Q^{2*} = (4.54, 4.54, 4.54, 4.54)$, $\gamma^* = (273.58, 273.58)$ and $\rho_3^* = (283.12, 283.12)$

Figure 3.3 illustrates the change of value of the merit function within the last fifteen iterations of the quasi-Newton algorithm in solving the modified example. It clearly indicates that the value of the merit function at the iterative point is almost equal to zero after 61 iterations. In other words, the above solution $\tilde{X}^*$ is indeed the solution of this example according to Proposition 3.1.

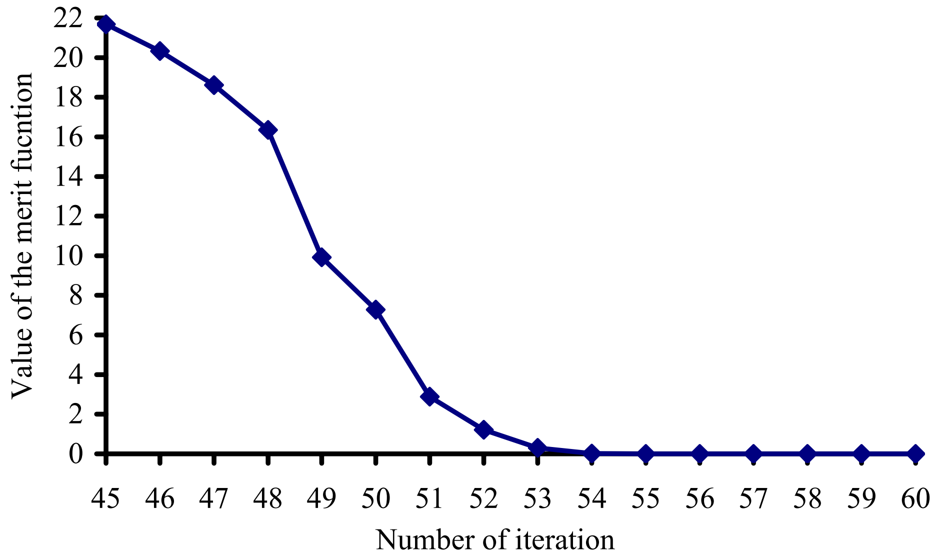


Figure 3.3 Change of value of merit function with respect to the number of iterations for the modified example

Since the second-order derivatives for production cost functions (3.45) and (3.46) are unbounded, Lipschitz continuity condition for the vector function $\tilde{F}\left(\tilde{X}\right)$ associated with this example does not hold. It thus means that the modified projection method may not be convergent for the example according to Theorem 4 of Nagurney et al. (2002). However, by trial and error on step size $\tilde{\alpha}$ in iterative scheme (3.38), it is found that find that when step size $\tilde{\alpha}$ in the effective interval (0, 0.005] the modified projection method is workable. Figure 3.4 evidences the convergent trend of the modified projection method with three different predetermined step sizes for the example. It can be seen that the modified projection method is divergent in the case of that step size $\tilde{\alpha} = 0.01$. In addition, the modified projection method will reach the stopping criterion (3.42) after 2315 iterations when step size $\tilde{\alpha} = 0.005$, but it will terminate after 9820 iterations when step size $\tilde{\alpha} = 0.001$. Hence, the performance of the modified projection method heavily depends on the value of the predetermined step size. Unfortunately, it is not an easy task to seek an appropriate step size for a SCNE problem.

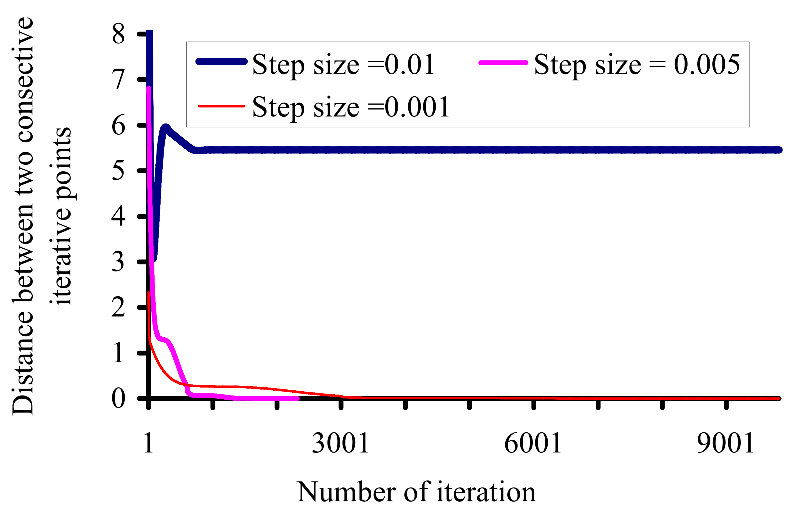


Figure 3.4 The convergent performance of the modified projection method

In terms of CPU times used by these two solution methods for the modified example, the quasi-Newton algorithm has spent 0.37 seconds, and the modified projection method with step size $\hat{\alpha} = 0.05$ has used 0.73 seconds. While both of these two numbers are acceptable in finding a solution, they explicitly imply that performance of the quasi-Newton method for this example is better than that of the modified projection method.

3.5.2 The other ten examples

Nagurney et al. (2002) provided four examples about the supply network equilibrium model with deterministic demands, and Dong et al. (2004) gave six examples for the supply network equilibrium model with random demands. They merely employed these ten examples to verify that the modified projection method is capable of solving their variational inequality formulations (3.7) and (3.14). The predetermined step sizes guaranteeing the convergence of the modified projection method expressed in eqns. (3.38)-(3.39) for these ten examples do theoretically exist. However, there is no practical guide to obtain these step sizes. By trial and error, an effective interval of the step size for each example can be estimated, which is tabulated in Tables 3.1 and 3.2, respectively. According to these two tables, it can be seen that the effective interval of the step size is varied over the ten examples.

With regard to these ten examples, both the quasi-Newton algorithm and the modified projection method with a step size in the corresponding effective interval shown in Tables 3.1 and 3.2 can generate the same solution. The CPU times they used for each example, however, are quite different. They can be compared by calculating ratio of the CPU time used by the quasi-Newton algorithm to the least CPU time used by the modified projection method, which are obtained by enumerating all possible effective step sizes. These ratios are listed in Tables 3.3 and 3.4. From these two tables, it can be seen that there are 6 cases out of the ten examples, among which the performance of the quasi-Newton algorithm is better than the modified projection method. If the predetermined step size in the modified projection method is equal to 0.01, the number of examples for which the CPU time used by the quasi-Newton is less than that used by the modified projection method will rise to 8.

Table 3.1 Effective intervals of step size $\tilde{\alpha}$ for the four examples in Nagurney et al. (2002)

Example 1	Example 2	Example 3	Example 4
(0, 0.06]	(0, 0.06]	(0, 0.04]	(0, 0.06]

Table 3.2 Effective intervals of step size $\hat{\alpha}$ for the six examples in Dong et al. (2004)

Example 1	Example 2	Example 3	Example 4	Example 5	Example 6
(0, 0.01]	(0, 0.02]	(0, 0.03]	(0, 0.03]	(0, 0.03]	(0, 0.02]

Table 3.3 Ratios of CPU time in seconds used by the quasi-Newton algorithm to the least CPU time used by the modified projection method for the four examples of Nagurney et al. (2004)

Example 1	Example 2	Example 3	Example 4
11.90	2.16	62.36	4.38

Table 3.4 Ratios of CPU time in seconds used by the quasi-Newton algorithm to the least CPU time used by the modified projection method for the six examples of Dong et al. (2004)

Example 1	Example 2	Example 3	Example 4	Example 5	Example 6
0.01	0.16	0.23	0.54	0.51	0.17

3.6 Discussion and Summary

This chapter has developed the unconstrained minimization formulation for the SCNE model with two cases: deterministic demands and random demands. It further shows that any stationary solution of the unconstrained minimization model derived is indeed the solution for the relevant SCNE model. In view of the difficulty that implementing the modified projection method must take an appropriate step size in advance, the well-known quasi-Newton algorithm is applied for solving the SCNE models. Numerical results from the benchmark examples not only reveal the limitation of the modified projection method but also confirm the advantage and flexibility of the quasi-Newton algorithm. Hence, this chapter has successfully provided the alternative formulation and solution method for the SCNE models. Finally, it should be pointed out that the idea of transferring VI formulations to unconstrained minimization problems can be not only used for the two SCNE models stated in this chapter, but also for all of the other SCNE models (Nagurney and Toyasaki, 2003, Nagurney and Toyasaki,2003, Nagurney and Toyasaki, 2005, Dong et al., 2005 and Nagurney and Matsypura, 2005) and even the other VI formulations defined on nonnegative orchant.

CHAPTER 4 COMPETITIVE FACILITY LOCATION ON DECENTRALIZED SUPPLY CHAINS

4.1 Introduction

This chapter is focused on a competitive facility location problem. When a firm locates a new manufacturing facility, and begins producing and shipping products to demand markets, it typically stimulates certain reactions in markets. For example, the introduction of a new facility increases the overall capacity of an industry, and hence can perturb the established economic equilibrium status of supplies, demands and product flows, which is actually a long-term steady market state due to competition. The introduction of this new capacity, and in the case of an “entering” firm, the introduction of an entirely new competitor on the market, will trigger some form of competitive response from existing firms in the industry. This would suggest that to truly make a profit maximizing location decision, the firm must anticipate the market’s reaction to a potential location decision, in its actual location decision-making process. It is this need to anticipate the market’s reaction that motivates the authors’ objective to develop facility location models that somehow include projected market reactions endogenously within the firm’s profit maximizing facility location objective function.

In this chapter, a SCNE model with production capacity constraints, which is an extension of the SCNE model developed by Nagurney et al. (2002), is proposed firstly. After successfully deriving the SCNE model with production capacity constraints, this chapter proceeds to build an MPEC model for a competitive facility location problem on decentralized supply chains. The proposed MPEC model involves binary (0-1) decision variables representing whether or not a candidate site is chosen and the parameterized VI constraint that formulates the SCNE model with the production capacity constraints in the case of a given feasible facility location solution.

4.2 Supply Chain Network Equilibrium Model with Production Capacity Constraints and Solution Method

For convenience, the notations and equations for the supply chain network equilibrium model introduced in Chapter 3 are continued to be used in this chapter.

4.2.1 Supply chain network equilibrium model with production capacity constraints

Let us consider the deterministic case of the SCNE model introduced in Chapter 3. In view of limited resources, such as financial budgets, equipment, space and available raw materials, managed or owned by manufacturers in the decentralized supply chain, it is more rational and practical to assume that quantities of the product produced by the manufacturers during a planning period should have upper bounds, namely,

$$\sum_{j=1}^{n} q_{ij} \le C_i, i = 1, \cdots, m \tag{4.1}$$

where C_i is the upper bound of the production level for manufacturer i. Inequalities expressed by (4.1) are cast as the production capacity constraints of manufacturers in this chapter.

Having taken the production capacity constraints into consideration, the set of all the feasible shipment patterns, i.e., shipment patterns that satisfy the constraints (3.4) and (4.1), for the decentralized supply chain can be expressed below.

$$\Omega = \left\{ \left(Q^1, Q^2\right) \in \Re_+^{mn+no} \left| \sum_{j=1}^{n} q_{ij} \leq C_i, i = 1, \cdots, m \text{ and } \sum_{k=1}^{o} q_{jk} \leq \sum_{i=1}^{m} q_{ij}, , j = 1, \cdots, n \right. \right\} \tag{4.2}$$

It is now ready to derive an SCNE model with production capacity constraints. Following the similar derivation in Nagurney et al. (2002), the SCNE conditions in the case of the production capacity constraints can be characterized as: finding a row vector $\left(Q^{1*}, Q^{2*}, \rho_3^*\right) \in \Omega \times \Re_+^o$ such that

$$\begin{aligned} &\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial f_i\left(Q^{1*}\right)}{\partial q_{ij}} + \frac{\partial c_{ij}\left(q_{ij}^*\right)}{\partial q_{ij}} + \frac{\partial c_j\left(Q^{1*}\right)}{\partial q_{ij}}\right] \times \left[q_{ij} - q_{ij}^*\right] \\ &+ \sum_{j=1}^{n}\sum_{k=1}^{o}\left[c_{jk}\left(Q^{2*}\right) - \rho_{3k}^*\right] \times \left[q_{jk} - q_{jk}^*\right] + \\ &\sum_{k=1}^{o}\left[\sum_{j=1}^{n} q_{jk}^* - d_k\left(\rho_3^*\right)\right] \times \left[\rho_{3k} - \rho_{3k}^*\right] \geq 0, \quad \forall \left(Q^1, Q^2, \rho_3\right) \in \Omega \times \Re_+^o \end{aligned} \tag{4.3}$$

Existence and uniqueness of the solution of VI (4.3) can be easily proved under the same assumptions in the SCNE model without considering the production capacity constraints by means of mathematical techniques used in Nagurney et al. (2002).

According to the Lagrangian duality for VIs (Auslender and Teboulle, 2000), there are optimal Lagrangian multipliers, λ_i^*, $i = 1, \cdots, m$, and γ_j^*, $j = 1, \cdots, n$, with respect to the production capacity constraints (4.1) and the stock capacity constraints (3.4), respectively, which defines set Ω as well, for the solution $\left(Q^{1*}, Q^{2*}, \rho_3^*\right)$ of the above VI. These nonnegative optimal Lagrangian multipliers together with the solution of VI (4.3) should fulfill the slackness complementary conditions:

$$\lambda_i^* \left(\sum_{j=1}^{n} q_{ij}^* - C_i \right) = 0, i = 1, \cdots, m \tag{4.4}$$

As such, the equilibrium price charged by a manufacturer for a retailer can be evaluated by

$$\rho_{1ij}^* = \frac{\partial f_i \left(Q^{1*} \right)}{\partial q_{ij}} + \frac{\partial c_{ij} \left(q_{ij}^* \right)}{\partial q_{ij}} + \lambda_i^*, i = 1, \cdots, m, j = 1, \cdots, n \tag{4.5}$$

Similarly, the equilibrium unit price of the product charged by a retailer can be calculated by

$$\rho_{2j}^* = \gamma_j^*, j = 1, \cdots, n \tag{4.6}$$

Eqns. (4.3)-(4.6) indicate that the SCNE model with production capacity constraints will be identical to the original SCNE model when the production capacity constraints become unbinding, namely, $\lambda_i^* = 0$, $i = 1, \cdots, m$. Because the set of feasible solutions for VI (4.3) , $\Omega \times \Re_+^o$, is a proper subset of the nonnegative orchant, the modified projection method suggested by Nagurney et al. (2002) is no longer available for solving VI (4.3). Thus, it urges us to seek an efficient and effective solution method for solving the SCNE model with production capacity constraints. Besides, the solution method is able to find the relevant optimal Lagrangian multipliers.

4.2.2 Logarithmic-quadratic proximal prediction-correction method

In the context of optimization, the classical proximal method replaces a minimization problem by a sequence of better behaved problems with a quadratic regularization term added to the objective function (Rockafellar, 1976). Many generalizations of this classical proximal method have been proposed recently after the success of interior point methods for linear programming. One of the main objectives is to replace the squared Euclidean norm by coercive regularizations that are able to implicitly deal with simple constraints such as box and linear constraints, giving raise to interior proximal methods (e.g., Auslender and Haddou, 1995; Teboulle, 1997).

Auslender et al. (1999) developed a logarithmic-quadratic proximal (LQP) method for monotone VIs defined on polyhedral sets, which is an interior proximal method with the global convergent property. However, the LQP method is merely an iterative solution framework without a tractable procedure in calculating an iterative point that is a solution of the inclusion equations generated at each iteration. He et al. (2006) successfully tackled such a flaw, and they created a notable LQP P-C method that includes the prediction and correction procedures to effectively solving the inclusion equations induced in LQP. Besides, the LQP P-C method can also attain the optimal Lagrangian multipliers. It thus fits all the requirements of a solution method for the SCNE model with the production capacity constraints. To customize the LQP P-C method for the SCNE model with the production capacity constraints, the LQP P-C method is elaborated below.

For the sake of presentation, VI (4.3) can be rewritten by vector notations as follows.

Find a row vector $\left(Q^{1*}, Q^{2*}, \rho_3^*\right) \in \Omega \times \Re_+^{o}$ such that

$$\begin{aligned}&F_1\left(Q^{1*}\right)\left(Q^1-Q^{1*}\right)^T+F_2\left(Q^{2*},\rho_3^*\right)\left(Q^2-Q^{2*}\right)^T\\&+F_3\left(Q^{2*},\rho_3^*\right)\left(\rho_3-\rho_3^*\right)^T\geq 0,\forall\left(Q^1,Q^2,\rho_3\right)\in\Omega\times\Re_+^o\end{aligned}\tag{4.7}$$

where the row vector functions:

$$F_1\left(Q^1\right)=\left(\cdots,\frac{\partial f_i\left(Q^1\right)}{\partial q_{ij}}+\frac{\partial c_{ij}\left(q_{ij}\right)}{\partial q_{ij}}+\frac{\partial c_j\left(Q^1\right)}{\partial q_{ij}},\cdots\right)\in\Re^{mn},\tag{4.8}$$

where $i=1,\cdots,m,\ j=1,\cdots,n$

$$F_2\left(Q^2,\rho_3\right)=\left(\cdots,c_{jk}\left(Q^2\right)-\rho_{3k},\cdots\right)\in\Re^{no},\ \text{where } j=1,\cdots,n,k=1,\cdots,o\tag{4.9}$$

$$F_3\left(Q^2,\rho_3\right)=\left(\cdots,\sum_{j=1}^{n}q_{jk}-d_k\left(\rho_3\right),\cdots\right)\in\Re^{o},\ \text{where } k=1,\cdots,o\tag{4.10}$$

In terms of vector notations, production capacity constraints (4.1) of the manufacturers and stock constraints (3.4) of retailers can be concisely rewritten as

$$Q^1A_1\leq C\tag{4.11}$$

$$Q^2A_2\leq Q^1A_3\tag{4.12}$$

where row vector $C=\left(\cdots,C_i,\cdots\right)\in\Re^m$ and matrices

$$A_1=\begin{bmatrix}I_1&\mathbf{0}&\cdots&\mathbf{0}\\\mathbf{0}&I_2&\cdots&\mathbf{0}\\\vdots&\mathbf{0}&\ddots&\vdots\\\mathbf{0}&\mathbf{0}&\cdots&I_m\end{bmatrix}_{mn\times m},A_2=\begin{bmatrix}J_1&\mathbf{0}&\cdots&\mathbf{0}\\\mathbf{0}&J_2&\cdots&\mathbf{0}\\\vdots&\mathbf{0}&\ddots&\vdots\\\mathbf{0}&\mathbf{0}&\cdots&J_n\end{bmatrix}_{no\times n}\text{ and }A_3=\begin{bmatrix}E_1\\\vdots\\E_m\end{bmatrix}_{mn\times n}\tag{4.13}$$

where column vectors: $I_1=\cdots=I_m=\left(1,1,\cdots 1\right)^T\in\Re^m$, $J_1=\cdots=J_n=\left(1,1,\cdots 1\right)^T\in\Re^n$, and all the E_i, $i=1,\cdots,m$, are identical to a $n\times n$ dimensional unit matrix .

Let $\|\cdot\|_\infty$ and $\|\cdot\|$ denote the infinity- and 2-norms of a vector, respectively. The LQP P-C method for solving VI (4.3) is elaborated below.

LQP P-C method

Step 0. (Initialization) Let parameters $\beta_0=1$, $v=1(>0)$, $\eta=0.9(<1)$, $\mu=0.1$, $\sigma=2.0>1$, an initial solution, $\left(Q^{1(0)},Q^{2(0)},\rho_3^{(0)}\right)\in\Re_+^{mn+no}\times\Re_+^o$, and initial

Lagrangian multipliers, $\lambda^{(0)} \in \Re_+^m$, $\gamma^{(0)} \in \Re_+^n$. Set the number of iteration $\tau = 0$.

Step 1. (Stopping criterion) If $\left\| e\left(Q^{1(\tau)}, Q^{2(\tau)}, \rho_3^{(\tau)}, \lambda^{(\tau)}, \gamma^{(\tau)}\right) \right\|_\infty \le \varepsilon$, in which ε is a predetermined tolerance, then stop; otherwise, continue; where vector $e\left(Q^{1(\tau)}, Q^{2(\tau)}, \rho_3^{(\tau)}, \lambda^{(\tau)}, \gamma^{(\tau)}\right) \in \Re^{mn+nk+o+m+n}$ defined as follows:

$$
\begin{aligned}
& e\left(Q^{1(\tau)}, Q^{2(\tau)}, \rho_3^{(\tau)}, \lambda^{(\tau)}, \gamma^{(\tau)}\right) = \\
& \Big(Q^{1(\tau)} - \left[Q^{1(\tau)} - \left(F_1\left(Q^{1(\tau)}\right) + \lambda^{(\tau)} A_1^T - \gamma^{(\tau)} A_3^T \right) \right]^+, \\
& Q^{2(\tau)} - \left[Q^{2(\tau)} - \left(F_2\left(Q^{2(\tau)}, \rho_3^{(\tau)}\right) + \gamma^{(\tau)} A_2^T \right) \right]^+, \\
& \rho_3^{(\tau)} - \left[\rho_3^{(\tau)} - F_3\left(Q^{2(\tau)}, \rho_3^{(\tau)}\right) \right]^+, \lambda^{(\tau)} - \left[\lambda^{(\tau)} + Q^{1(\tau)} A_1 - C \right], \\
& \gamma^{(\tau)} - \left[\gamma^{(\tau)} + Q^{2(\tau)} A_1 - Q^{1(\tau)} A_3 \right]^+ \Big)
\end{aligned}
\tag{4.14}
$$

where $[y]^+$ is the projection operator of vector y onto the nonnegative orthant. Namely, assuming that $y = (y_1, \cdots, y_N) \in \Re^N$, a vector of N-dimensional real space, then $[y]^+ = \left(y_1^+, \cdots, y_N^+\right) \in \Re_+^N$ with elements:

$$
y_l^+ = \begin{cases} y_l, & \text{if } y_l \ge 0 \\ 0, & \text{otherwise} \end{cases}, l = 1, \cdots, N \tag{4.15}
$$

Step 2. (Prediction step) Produce a predictor, $\left(\hat{Q}^{1(\tau)}, \hat{Q}^{2(\tau)}, \hat{\rho}_3^{(\tau)}, \hat{\lambda}^{(\tau)}, \hat{\gamma}^{(\tau)}\right)$, that well approximates a solution of the inclusion equations, by executing the efficient manipulations:

Step 2.1. Choose a predictor in light of step size β_τ as follows.

$$
\hat{\lambda}^{(\tau)} = \left[\hat{\lambda}^{(\tau)} + \frac{\beta_k}{\nu}\left(Q^{1(\tau)} A_1 - C \right) \right]^+ \tag{4.16}
$$

$$
\hat{\gamma}^{(\tau)} = \left[\hat{\gamma}^{(\tau)} + \frac{\beta_k}{\nu}\left(Q^{2(\tau)} A_2 - Q^{1(\tau)} A_3 \right) \right]^+ \tag{4.17}
$$

$$
\hat{q}_{ij}^{(\tau)} = \left(s_{ij} + \sqrt{\left(s_{ij}\right)^2 + 4\mu\left(q_{ij}^{(\tau)}\right)^2} \right) / 2, i = 1, \cdots, m, j = 1, \cdots, n \tag{4.18}
$$

$$\hat{q}_{jk}^{(\tau)} = \left(s_{jk} + \sqrt{\left(s_{jk}\right)^2 + 4\mu\left(q_{jk}^{(\tau)}\right)^2} \right) / 2,\ j = 1,\cdots,n, k = 1,\cdots,o \tag{4.19}$$

$$\hat{\rho}_{3k}^{(\tau)} = \left(t_k + \sqrt{\left(t_k\right)^2 + 4\mu\left(\rho_{3k}^{(\tau)}\right)^2} \right) / 2 \tag{4.20}$$

where s_{ij}, $i = 1,\cdots,m$, $j = 1,\cdots,n$, s_{jk}, $j = 1,\cdots,n$, $k = 1,\cdots,o$ and t_k, $k = 1,\cdots,o$ are the elements of the following three vectors:

$$\left(\cdots, s_{ij} \cdots\right) = \left(1-\mu\right) Q^{1(\tau)} - \beta_\tau \left(F_1\left(Q^{1(\tau)}\right) + \hat{\lambda}^{(\tau)} A_1^T - \hat{\gamma}^{(\tau)} A_3^T \right) \in \Re^{mn} \tag{4.21}$$

$$\left(\cdots, s_{jk} \cdots\right) = \left(1-\mu\right) Q^{2(\tau)} - \beta_\tau \left(F_2\left(Q^{2(\tau)}, \rho_3^{(\tau)}\right) + \hat{\gamma}^{(\tau)} A_2^T \right) \in \Re^{nk} \tag{4.22}$$

$$\left(\cdots, t_k, \cdots\right) = \left(1-\mu\right) \rho_3^{(\tau)} - \beta_\tau F_3\left(Q^{2(\tau)}, \rho_3^{(\tau)}\right) \in \Re^{o} \tag{4.23}$$

Step 2.2. Seek a step size, β_τ, fulfilling a certain condition

Let us calculate

$$r_\tau = \sqrt{\frac{\nu\left\|\xi_1^{(\tau)}\right\|^2 + \left(1+\mu\right)\left\|\xi_2^{(\tau)}\right\|^2}{\nu\left(1-\mu^2\right)\left\|\pi_1^{(\tau)}\right\|^2 + \nu^2\left(1-\mu\right)\left\|\pi_2^{(\tau)}\right\|^2}} \tag{4.24}$$

where vectors $\xi_1^{(\tau)} \in \Re^{mn+nk+o}$, $\xi_2^{(\tau)} \in \Re^{m+n}$, $\pi_1^{(\tau)} \in \Re^{mn+nk+o}$ and $\pi_2^{(\tau)} \in \Re^{m+n}$ with

$$\begin{aligned}\xi_1^{(\tau)} = \beta_\tau \Big[&\left(F_1\left(\hat{Q}_1^{(\tau)}\right), F_2\left(\hat{Q}_2^{(\tau)}, \hat{\rho}_3^{(\tau)}\right), F_3\left(\hat{Q}_2^{(\tau)}, \hat{\rho}_3^{(\tau)}\right)\right) - \\ &\left(F_1\left(Q_1^{(\tau)}\right), F_2\left(Q_2^{(\tau)}, \rho_3^{(\tau)}\right), F_3\left(Q_2^{(\tau)}, \rho_3^{(\tau)}\right)\right) \Big]\end{aligned} \tag{4.25}$$

$$\xi_2^{(\tau)} = \beta_\tau \left[\left(Q^{1(\tau)} - \hat{Q}^{1(\tau)}\right) A_1, \left(Q^{2(\tau)} - \hat{Q}^{2(\tau)}\right) A_2 - \left(Q^{1(\tau)} - \hat{Q}^{1(\tau)}\right) A_3 \right] \tag{4.26}$$

$$\pi_1^{(\tau)} = \left(Q^{1(\tau)}, Q^{2(\tau)}, \rho_3^{(\tau)}\right) - \left(\hat{Q}^{1(\tau)}, \hat{Q}^{2(\tau)}, \hat{\rho}_3^{(\tau)}\right) \tag{4.27}$$

$$\pi_2^{(\tau)} = \left(\lambda^{(\tau)}, \gamma^{(\tau)}\right) - \left(\hat{\lambda}^{(\tau)}, \hat{\gamma}^{(\tau)}\right) \tag{4.28}$$

If $r_\tau > \eta$, then reduce β_τ by setting $\beta_\tau = \beta_\tau \times 0.8 / r_k$, and go to Step 2.1. Otherwise, go to Step 3.

Step 3. (Adjust step size β_τ and parameter ν for the next iteration if necessary).

Enlarge β_τ for the next iteration if r_τ is small enough by

$$\beta_{\tau+1} = \begin{cases} \beta_\tau \times 0.7 / r_\tau, \text{ if } r_\tau \le 0.5 \\ \beta_\tau, \text{otherwise} \end{cases} \tag{4.29}$$

Reduce or increase parameter ν according to the formula:

$$\nu = \begin{cases} 0.5\nu, \text{if } \left\|\xi_1^{(\tau)}\right\| / \sqrt{1+\mu} > 4\left\|\xi_2^{(\tau)}\right\| / \sqrt{\nu} \\ 2\nu, \text{if } 4\left\|\xi_1^{(\tau)}\right\| / \sqrt{1+\mu} < \left\|\xi_2^{(\tau)}\right\| / \sqrt{\nu} \\ \nu, \text{otherwise} \end{cases} \tag{4.30}$$

Step 4. (Calculate the step size of the correction step). Set the step size,

$$\alpha_\tau = \sigma\alpha_\tau^*\beta_\tau\left(\frac{1-\mu}{1+\mu}\right) \tag{4.31}$$

where

$$\alpha_\tau^* = \frac{\pi_1^{(\tau)}\left(\pi_1^{(\tau)} + \xi_1^{(\tau)}\right)^T + \pi_2^{(\tau)}\left(\nu\pi_2^{(\tau)} + \xi_2^{(\tau)}\right)^T}{\left[\pi_1^{(\tau+1)} + \xi_1^{(\tau)} / (1+\mu)\right]\left[(1+\mu)\pi_1^{(\tau+1)} + \xi_1^{(\tau)}\right]^T + \left(\pi_2^{(\tau)} + \xi_2^{(\tau)} / \nu\right)\left(\nu\pi_2^{(\tau)} + \xi_2^{(\tau)}\right)^T} \tag{4.32}$$

Step 5. (Correction step) Calculate the new iterative solution, $\left(Q^{1(\tau+1)}, Q^{2(\tau+1)}, \rho_3^{(\tau+1)}, \lambda^{(\tau+1)}, \gamma^{(\tau+1)}\right)$, by performing the operations:

$$\lambda^{(\tau+1)} = \left[\lambda^{(\tau)} + \frac{\alpha_\tau}{\nu}\left(\hat{Q}^{1(\tau)}A_1 - C\right)\right]^+ \tag{4.33}$$

$$\gamma^{(\tau+1)} = \left[\gamma^{(\tau)} + \frac{\alpha_\tau}{\nu}\left(\tilde{Q}^{1(\tau)}A_1 - \tilde{Q}^{2(\tau)}A_3\right)\right]^+ \tag{4.34}$$

$$q_{ij}^{(\tau+1)} = \left(\hat{s}_{ij} + \sqrt{\left(\hat{s}_{ij}\right)^2 + 4\mu\left(q_{ij}^{(\tau)}\right)^2}\right) / 2, i = 1, \cdots, m, j = 1, \cdots, n \tag{4.35}$$

$$q_{jk}^{(\tau+1)} = \left(\hat{s}_{jk} + \sqrt{\left(\hat{s}_{jk}\right)^2 + 4\mu\left(q_{jk}^{(\tau)}\right)^2}\right) / 2, j = 1, \cdots, n, k = 1, \cdots, o \tag{4.36}$$

$$\rho_{3k}^{(\tau+1)} = \left(\hat{t}_k + \sqrt{\left(\hat{t}_k\right)^2 + 4\mu\left(\rho_{3k}^{(\tau)}\right)^2}\right) / 2 \tag{4.37}$$

where $\hat{s}_{ij}, i = 1,\cdots,m$, $j = 1,\cdots,n$, $\hat{s}_{jk}$, $j = 1,\cdots,n$, $k = 1,\cdots,o$ and $\hat{t}_k$, $k = 1,\cdots,o$ are the elements of the three vectors as follows.

$$\left(\cdots,\hat{s}_{ij}\cdots\right) = \left(1-\mu\right)Q^{1(\tau)} - \alpha_\tau\left(F_1\left(\hat{Q}^{1(\tau)}\right) + \hat{\lambda}^{(\tau)}A_1^T - \hat{\gamma}^{(\tau)}A_3^T\right) \in \Re^{mn} \tag{4.38}$$

$$\left(\cdots,\hat{s}_{jk}\cdots\right) = \left(1-\mu\right)Q^{2(\tau)} - \alpha_\tau\left(F_2\left(\hat{Q}^{2(\tau)},\hat{\rho}_3^{(\tau)}\right) + \hat{\gamma}^{(\tau)}A_2^T\right) \in \Re^{nk} \tag{4.39}$$

$$\left(\cdots,\hat{t}_k,\cdots\right) = \left(1-\mu\right)\rho_3^{(\tau)} - \alpha_\tau F_3\left(\hat{Q}^{2(\tau)},\hat{\rho}_3^{(\tau)}\right) \in \Re^{o} \tag{4.40}$$

Let $\tau = \tau + 1$, and go to Step 1.

It has already been recognized that $\left(Q^{1*}, Q^{2*}, \rho_3^*, \lambda^*, \gamma^*\right)$ is the solution of VI (4.3) if and only if $\left\|e\left(Q^{1*}, Q^{2*}, \rho_3^*, \lambda^*, \gamma^*\right)\right\|_\infty = 0$ (He et al, 2004 or 2006). Hence, the stopping criterion in Step 1 adopted by the LQP P-C method is rational. He et al. (2006) demonstrated that the procedure comprising Steps 2.1 and 2.2 to obtain an appropriate step size β_τ will be terminated after limited iterations. They further rigorously proved the convergence of the LQP P-C method for any monotone VI defined on a polyhedral set.

With respect to the LQP P-C method, the computational burden of manipulations shown in eqns. (4.16)-(4.40) is very tiny because it merely needs to perform fundamental mathematical operations such as comparisons, additions, multiplications and square-root. Other than that, Step 3 that aims to automatically adjust the step size and the algorithmic parameter according to changes of some merit indices will make the method more effective and robust.

Finally, it should be pointed out that this idea and solution method of SCNE model with capacity constraints can also be used in other SCNE models except for the SCNE model developed by Nagurney and her colleagues in 2002.

4.3 MPEC Model for Competitive Facility Location Problem

The competitive facility location problem on a decentralized supply chain is a strategic level decision problem of a firm. It is to maximize an objective such as the firm's profit through determining locations of its facilities as well as the production levels of these facilities, taking into account the market competition existing in the decentralized supply chain which can be captured by the SCNE model with production capacity constraints.

Suppose that there are m manufacturers, n retailers and o demand markets for an existing decentralized supply chain. All of these manufacturers produce substitutable products and supply them to the demand markets via the retailers. Let L be the number of candidate sites where the entering firm's facilities can be built, and they are numbered by $1, 2, \cdots, L$, respectively. Moreover, it is assumed that the newly built facilities are treated as additional manufacturers joining the existing decentralized supply chain, and consequently competition arises among these new manufacturers as well as the existing manufacturers. It is further assumed that the entering firm is able to predict SCNE shipment and prices patterns between the manufacturers and the retailers after the new facilities joining the decentralized supply chain. Note that the latter assumption is available for the situation in which the existing market is made up of a large number of small firms; by contrast, the entering firm is a large one with the ability to influence market prices (Miller et al., 1992). It should be pointed out that these two assumptions were made by Tobin and Friesz (1986) on studying the competitive facility location problem with the SPE constraints.

Let x_l be the binary decision variable to represent whether or not location l will be selected by the entering firm, namely:

$$x_l = \begin{cases} 1, & \text{a facility is built at site } l \\ 0, & \text{otherwise} \end{cases}, \; l = 1, \cdots, L \tag{4.41}$$

Let C_l be the production capacity of the facility located at candidate site l ($l = 1, 2...L$). The entering firm's objective should be a function of the SCNE shipment and prices patterns between manufacturer and retailers of the decentralized supply chain involving the new facilities. To distinguish notations of decision variables used in Sections 4.2.1 and 4.2.2, let $x = (\cdots, x_l, \cdots)$ be a row vector of all the binary decision variable; $\tilde{Q}^1(x) = (\cdots, \tilde{q}_{ij}^1(x), \cdots)$ and $\tilde{\rho}_1(x) = (\cdots, \tilde{\rho}_{1ij}(x), \cdots)$ be the row vectors of all the SCNE shipments and prices between manufacturers, $i \in M(x)$, and retailers, $j = 1, \cdots, n$, where $M(x)$ is set of all manufacturers associated with a decision variable *x*, namely

$$M(x) = \{ l | x_l = 1, l = 1, \cdots, L \} \cup \{ L+1, \cdots, L+m \} \tag{4.42}$$

In addition, let $h\left(x, \tilde{Q}^1(x), \tilde{\rho}_1(x)\right)$ denote the generic objective function concerned in the competitive facility location problem.

The competitive facility location problem sometimes has to consider a few constraints such as a limited investment budget. Without loss of generality, it is assumed that there are P constraints denoted by functions, $g_p\left(x, \tilde{Q}^1(x), \tilde{\rho}_1(x)\right)$, $p = 1, 2, \cdots, P$. Obviously, $P = 0$ means that there is no constraint. Therefore, the competitive facility location problem with the SCNE constraints can be formulated by the MPEC model:

$$\max_{x} h\left(x, \tilde{Q}^1(x), \tilde{\rho}_1(x)\right) \tag{4.43}$$

subject to

$$g_p\left(x, \tilde{Q}^1(x), \tilde{\rho}_1(x)\right) \le 0,\ p = 1, \cdots, P \tag{4.44}$$

$$\begin{aligned}
&\sum_{i \in M(x)} \sum_{j=1}^{n} \left[\frac{\partial f_i\left(\tilde{Q}^1(x)\right)}{\partial q_{ij}} + \frac{\partial c_{ij}\left(\tilde{q}_{ij}(x)\right)}{\partial q_{ij}} + \frac{\partial c_j\left(\tilde{Q}^1(x)\right)}{\partial q_{ij}} \right] \times \left[q_{ij} - \tilde{q}_{ij}(x) \right] + \\
&\sum_{j=1}^{n} \sum_{k=1}^{o} \left[c_{jk}\left(Q^2(x)\right) - \rho_{3k}(x) \right] \times \left[q_{jk} - q_{jk}(x) \right] + \\
&\sum_{k=1}^{o} \left[\sum_{j=1}^{n} q_{jk}(x) - d_k\left(\rho_3(x)\right) \right] \times \left[\rho_{3k} - \rho_{3k}(x) \right] \ge 0 \\
&,\ \forall \left(Q^1, Q^2, \rho_3\right) \in \Omega(x) \times R_+^o
\end{aligned} \tag{4.45}$$

$$x_l = 0 \text{ or } 1 \quad l = 1, \cdots, L \tag{4.46}$$

where $\Omega(x)$, the set of all feasible shipment patterns for the decentralized supply chain involving the new built facilities , can be expressed by

$$\Omega(x) = \left\{ \left(Q^1, Q^2\right) \in \Re_+^{m_x n + no} \left| \sum_{j=1}^{n} q_{ij} \le C_i, i \in M(x), \sum_{k=1}^{o} q_{jk} \le \sum_{i \in M(x)} q_{ij}, j = 1, \cdots, n \right. \right\} \tag{4.47}$$

where m_x is the cardinality of set $M(x)$ defined by eqn. (4.42). Note that $\tilde{\rho}_1(x) = \left(..., \tilde{\rho}_{1ij}(x), ...\right)$, which is a row vector of $\tilde{\rho}_{1ij}(x)$, involved in eqns. (4.43)-(4.44) has the following analytical expressions according to eqn. (4.5).

$$\tilde{\rho}_{1ij}(x) = \frac{\partial f_i\left(\tilde{Q}^1(x)\right)}{\partial q_{ij}} + \frac{\partial c_{ij}\left(\tilde{q}_{ij}(x)\right)}{\partial q_{ij}} + \lambda_i(x), i \in M(x),\ j = 1, \cdots, n \tag{4.48}$$

where $\lambda_i(x)$, $i \in M(x)$, is the optimal Lagrangian multiplier of the parameterized VI (4.45).

It can be seen that the above MPEC model is an integer programming-like optimization problem. However, its constraints include VI (4.45) that describes the SCNE conditions with the production capacity constraints associated with the decision variable *x*. Note that the MPEC model (4.43)-(4.46) is more generic. It can be customized by specifying the objective function (4.43) and constraints (4.44) according to different requirements in a competitive facility location problem. The following two instances demonstrate the flexibility of the above MPEC model.

Instance 1. The profit maximization with budget constraints

Most facility location problems aim to maximize the profit subject to limited budget. Let F_l be the fixed cost of setting up a facility with certain production capacity at site l, $l = 1, \cdots, L$, and B be the maximum investment budget of the entering firm. Hence, the objective function of the preceding MPEC model for the competitive facility location problem with budget constraints can be specified below.

$$\max h\left(x, \tilde{Q}^1(x), \tilde{\rho}_1(x)\right) = \sum_{i \in M_1(x)} \sum_{j=1}^{n} \left[\tilde{\rho}_{1ij}(x) \tilde{q}_{lj}(x) - c_{ij}\left(\tilde{Q}^1(x)\right) - f_i\left(\tilde{Q}^1(x)\right) \right] - \sum_{l=1}^{L} x_l F_l \tag{4.49}$$

where $M_1(x)$, the set of facility locations chosen by the entering firm, is defined by

$$M_1(x) = \left\{ l \middle| x_l = 1, l = 1, \cdots, L \right\} \tag{4.50}$$

The first term in the right hand side of eqn. (4.49) is the revenue the entering firm can gain from the built facilities, and the send term is the total cost of setting up these facilities. In addition, the budget constraint can be expressed by

$$\sum_{l=1}^{L} x_l F_l \le B \tag{4.51}$$

Instance 2. The return ratio maximization

Maximizing a return ratio such as return on logistics assets (ROLA) is also an important objective adopted in facility location decisions (Ballou, 2001). In this case, there is no restriction on the entering firm's budget. However, the objective has become maximization of the return ratio. Hence, the objective function (4.43) of the MPEC model has the following analytical expression:

$$\max h\left(x,\tilde{Q}^1(x),\tilde{\rho}_1(x)\right)=\frac{\sum_{i\in M_1(x)}\sum_{j=1}^{n}\left[\tilde{\rho}_{1ij}(x)\tilde{q}_{lj}(x)-c_{ij}\left(\tilde{Q}^1(x)\right)-f_i\left(\tilde{Q}^1(x)\right)\right]}{\sum_{l=1}^{L}x_l F_l} \tag{4.52}$$

4.4 Solution Algorithm

The preceding competitive facility location problem is an NP-complete problem. The well-known branch-and-bound or branch-and-cut algorithm for integer programming problems is no longer available for solving the above MPEC model due to the parameterized VI (4.45). The method of exhaustive enumeration is able to find the optimal solution of the MPEC model. Nonetheless, it can only solve the small-scale problems due to computer capacity. Hence, GA as one of the well-recognized meta-heuristic method can be adopted for solving the MPEC model.

GAs start with a population of individuals represented by chromosomes. Chromosomes from one population are taken and used to form a new generation of population according to their fitness – the more suitable they are, the more chance they can be selected to reproduce. Encoding of chromosome and choice of fitness function heavily depend on the nature of an optimization problem. With respect to the forgoing MPEC model, a chromosome is encoded by the row vector of binary decision variables, namely, $x=(x_1,\cdots,x_L)$. Apart from the parameterized VI (4.45), the MPEC model has another group of constraints shown in eqn. (4.44) that leads to the computational difficulty for GA to check the feasibility of a chromosome.

Fortunately, the penalty function approach arising from the nonlinear programming (Bazaraa et al., 1993) can be employed to tackle the problem. In other words, the fitness function of the GA is defined by the following function including a penalty term related to the constraint (4.44):

$$H(x) = h\left(x, \tilde{Q}^1(x), \tilde{\rho}_1(x)\right) - \omega \sum_{p=1}^{P} \max\left\{0, g_p\left(x, \tilde{Q}^1(x), \tilde{\rho}_1(x)\right)\right\} \tag{4.53}$$

where ω is a suitable positive penalty parameter such that the function value of $H(x)$ at any infeasible solution is less than that at any feasible solution. Having transformed the constraint (4.44) into the objective function by using a penalty term, given the chromosome x, the parameterized VI (4.45) can be solved by the LQP P-C method.

A conventional GA consists of three main steps: selection, crossover and mutation. The selection step attends to select chromosomes from the population to be parents for crossover. The stronger the chromosomes, i.e., chromosomes with the better fitness function values, the higher the probability which will be chosen to be the parents to generate a new chromosome. Note that there are a few schemes, including roulette wheel selection (Golberg, 1989), rank selection (Grefenstette and Baker, 1989), Boltzman selection (Goldberg, 1990) and tournament selection (Goldberg and Deb, 1991), to select the better chromosomes. The crossover step is the process of combining the genes of a selected chromosome with those of another to create offspring that inherit traits of both parents. As the binary encoding scheme of a chromosome is chosen in this study, several crossover methods such as single point crossover, two point crossover over, uniform crossover and arithmetic crossover can be used. The mutation step is a gene operator used to maintain genetic diversity from one generation of a population of chromosome to the next. It is analogous to biological mutation.

Compared to the classical GA, the LQP P-C method is necessitated when evaluating the fitness function (4.53) for a given chromosome. The GA incorporating the LQP P-C method for solving the MPEC is called the hybrid GA-LQP P-C, which is stated below.

Hybrid GA-LQP P-C

Step 0. (Initialization) Randomly generate a population of N chromosomes.

Step 1. (Calculation of the fitness function) For each chromosome x in the population, the value of fitness function $F(x)$ defined by eqn. (4.53) is evaluated after implementing the LQP-PC method for the parameterized VI (4.45) associated with the chromosome.

Step 2. (Generation of a new temporary population). Repeat the following three sub-steps until the new population is completed.

Step 2.1. (Selection) According to the fitness function values evaluated in Step 1, use the roulette wheel selection method to choose two parent chromosomes from the population.

Step 2.2. (Crossover) With a crossover probability, denoted by r_{cr}, cross over the parents to form a new offspring according to the one point crossover method. If no crossover is performed, offsprings are the exact copy of the parents

Step 2.3. (Mutation) Generate a number from [0,1] for each gene of the new offspring. If this number is less than the mutation probability, denoted by r_{mu}, change the value of this gene from 0 to 1 or vice versa.

Step 3. Select the best N chromosomes from the new temporary population and the last population, which forms the new population for the next generation.

Step 4. (Stopping criterion). If a stopping criterion is fulfilled, then terminate, and output the best solution from the population. Otherwise, go to Step 1.

For the above hybrid GA, Step 3 is an elitism strategy (Eshelman, 1991) by copying the strongest N chromosomes from the new temporary population and the old population to generate a new generation. This guarantees monotonic non-degradation of the best solution from an old population to a new population. There are two termination criteria that can be adopted in Step 4. It can be terminated when it achieves maximum number of generations specified or if there is no improvement in the fitness function value of the strongest chromosome in the population for more than the number of generations specified. Note that the performance of the hybrid GA-LQP P-C may depend on the population size and crossover and mutation probabilities.

4.5 Numerical Examples

To demonstrate the preceding models and solution methods, this chapter proceeds to carry out three numerical examples. The first one intends to numerically show a distinction between the SCNE models with and without the production capacity constraints, and to exhibit strength of the LQP P-C method. The second and third examples target at the MPEC model of the competitive facility location problem and the hybrid GA incorporated with the LQP P-C method.

4.5.1 An example for supply chain network equilibrium model with the production capacity constraints

Example 4 of Nagurney et al. (2002) is here taken as our numerical example, and it is further assumed that each manufacturer has the production capacity tabulated in Table 4.1.

Table 4.1 Production capacity of each Manufacturer

Manufacturer (i)	1	2	3
Production Capacity (C_i)	20	30	100

Referring to the LQP P-C method, the number of iterations required in the prediction step and the step size adjustment scheme of the correction step heavily depend on parameters η and σ, respectively. Figure 4.1 shows performance of the LQP P-C method with three different groups of values of parameters η and σ after 20 iterations when solving the numerical example for the SCNE with the production capacity constraints.

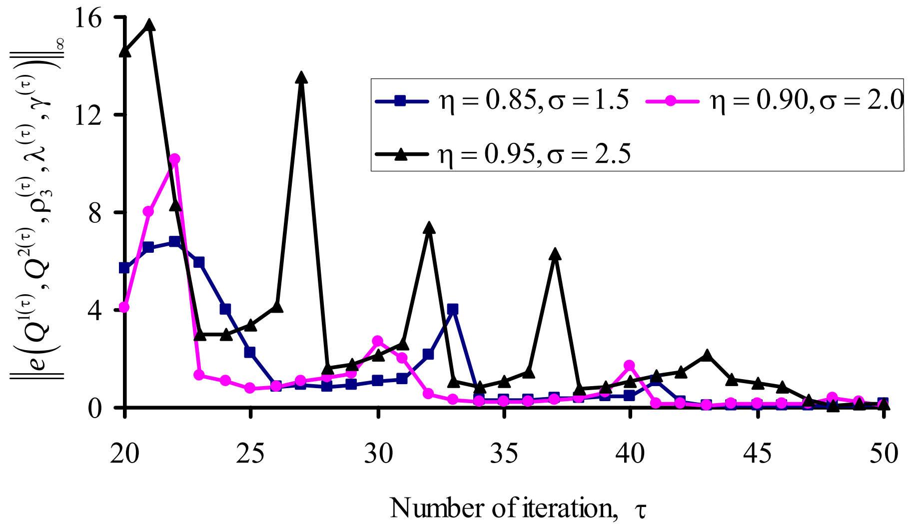


Figure 4.1 The convergent performance of the LQP P-C method with different parameters

Note that values of $\left\| e\left(Q^{1(\tau)}, Q^{2(\tau)}, \rho_3^{(\tau)}, \lambda^{(\tau)}, \gamma^{(\tau)}\right) \right\|_\infty$ within twenty iterations are not presented here due to their sharp changes in terms of magnitude. This figure confirms the global convergent property of the LQP P-C method for different parameters because $\left\| e\left(Q^{1(\tau)}, Q^{2(\tau)}, \rho_3^{(\tau)}, \lambda^{(\tau)}, \gamma^{(\tau)}\right) \right\|_\infty$ approaches zero with the increasing number of iterations.

Implementing the LQP P-C method for the example yields the solutions of the SCNE model with and without the production capacity constraints, including the optimal Lagrangian multipliers, which are listed in Table 4.2. According to Table 4.2, it can be observed that the production capacity constraint, $q_{11}^* + q_{12}^* \leq 20$, makes the solution q_{11}^* and q_{12}^* of the SCNE model and SCNE with production capacity constraints different. This suggests that the production capacity constraints do affect the decisions of the manufacturers and the equilibrium state of the supply chain.

Table 4.2 Solutions of the supply chain network equilibrium models with and without production capacity constraints

Solution	With capacity constraints	Without capacity constraints
Shipments between Manufacturers and Retailers	$q_{11}^* = q_{12}^* = 10.000$	$q_{11}^* = q_{12}^* = 12.395$
	$q_{21}^* = q_{22}^* = 13.077$	$q_{21}^* = q_{22}^* = 12.395$
	$q_{31}^* = q_{32}^* = 50.000$	$q_{31}^* = q_{32}^* = 50.078$
Shipments between Retailers and Demand markets	$q_{11}^* = q_{12}^* = 24.359$	$q_{11}^* = q_{12}^* = 24.956$
	$q_{21}^* = q_{22}^* = 24.359$	$q_{21}^* = q_{22}^* = 24.956$
	$q_{31}^* = q_{32}^* = 24.359$	$q_{31}^* = q_{32}^* = 24.956$
Optimal Lagrangian multipliers with respect to the production capacity constraints	$\lambda_1^* = 27.7015$ $\lambda_2^* = 0$ $\lambda_3^* = 5.3574$	N.A.
Optimal Lagrangian multipliers with respect to the stock capacity constraints	$\gamma_1^* = 242.435$ $\gamma_2^* = 242.435$	$\gamma_1^* = 241.496$ $\gamma_2^* = 241.496$
Prices consumers at different demand makers are willing to pay	$\rho_{31}^* = \rho_{32}^* = \rho_{33}^* = 271.794$	$\rho_{31}^* = \rho_{32}^* = \rho_{33}^* = 271.454$

4.5.2 An example for analyzing impact of the production capacity and budget in the MPEC model

The competitive facility location problem adopted here belongs to the preceding instance 1, i.e., it wishes to maximize the profit subject to the budget constraint. It is assumed that the existing decentralized supply chain consists of 5 manufacturers, 10 retailers and 10 demand markets, and that there are 10 candidate facility locations from which the entering firm can choose, namely, $m = 5$, $n = 10$, $o = 10$ and $L = 10$.

The data for this example is constructed for easy interpretation purposes. Table 4.3 gives the fixed setting up cost and two sets of production capacities of a facility that will be built at a candidate location. It is also assumed that the available budget of the entering firm is 200.0, i.e., $B = 200.0$.

Following the production functions used in the numerical examples by Nagurney et al. (2002) and Dong et al. (2004), the production cost functions for the manufacturers including the new joining ones are assumed to have the form:

$$f_i\left(Q^1\right) = 0.0005\left(\sum_{j=1}^{10} q_{ij}\right)^2 + i\left(\sum_{j=1}^{10} q_{ij}\right), i = 1, \cdots, 15 \tag{4.54}$$

The transaction cost functions between manufacturers and retailers, and the handling cost functions of the retailers are defined below.

$$c_{ij}\left(Q^1\right) = 2q_{ij}^2 + \left(i + j\right)q_{ij}, i = 1, \cdots, 15, j = 1, \cdots, 10 \tag{4.55}$$

$$c_j\left(Q^1\right) = 2\left(\sum_{i=1}^{15} q_{ij}\right)^2, j = 1, \cdots, 10 \tag{4.56}$$

The transaction cost functions between the retailers and the demand markets have the expressions:

$$c_{jk}\left(Q^2\right) = q_{jk} + 5, j = 1, \cdots, 10, k = 1, \cdots, 10 \tag{4.57}$$

The demand functions at the demand markets take the form:

$$d_k\left(\rho_3\right) = -5\rho_{3k} - 2\sum_{k=1}^{15} \rho_{3k} + 2000, k = 1, \cdots, 10 \tag{4.58}$$

Table 4.3 Production capacities and setting up costs of facilities located at candidate locations

Location candidate	First set of production capacities	Second set of production capacities	Setting up cost
1	20	13	55
2	18	12	52
3	18.4	18.4	57
4	25	16	58
5	16.6	25	50
6	18.5	18.8	52
7	16	22	56
8	17	17	48
9	20	17.2	55
10	23	18.8	58

As there are ten candidate locations only, all the solutions of the facility location, including infeasible ones, amount to $2^{10} = 1024$. Therefore, the exhaustive enumeration method incorporating with the LQP P-C method can be employed to find the optimal solution of the MPEC model of this example. Let values of the parameters necessitated in the LQP P-C method be $\eta = 0.9$, $\sigma = 2.0$ and $\varepsilon = 10^{-8}$ (tolerance in the stopping criterion). For three production capacity scenarios: two sets of the production capacities, shown in Table 4.3, and unlimited production capacities, performing the exhaustive enumeration method for the MPEC model of the numerical example comes out with the maximal profits and the optimal facility locations, which are presented in Table 4.4.

Table 4.4 Maximal profits and the optimal solutions of the MPEC model with different production capacity scenarios

Scenario	Maximal profit	Optimal facility locations
First set of production capacities	1054.189	Location 1,2 and 4
Second set of production capacities	867.530	Locations 1,3 and 5
Unlimited production capacities	991.468	Locations 1 and 2

Table 4.4 indicates that the maximal profit subject to the first set of the production capacities is higher than that subject to the second set of the production capacities. This is because the overall production capacity of the decentralized supply chain induced by the first set of the production capacities is greater than that induced by the second set of the production capacities. In addition, it is very interesting to see that the maximal profit subject to the first set of the production capacities is also higher than that without any production capacity constraint (i.e., unlimited production capacity). This is because the market competition in the case of limited production capacity that will raise the supply prices between manufacturers and retailers.

With regard to the competitive facility location problem on a decentralized supply chain, the maximal profit and total expenditure spent by the entering firm in setting up manufacturing facilities should vary with the budget. Figures 4.2 and 4.3 depict changes of these two indices with different budgets, respectively, for this numerical example. According to these two figures, it can be seen that both the maximal profit and the total expenditure become a constant for budget levels greater than 240. In other words, more budget may not lead to more profit due to free-market competition.

It implicitly verifies the economic principle of the Nash noncooperative game.

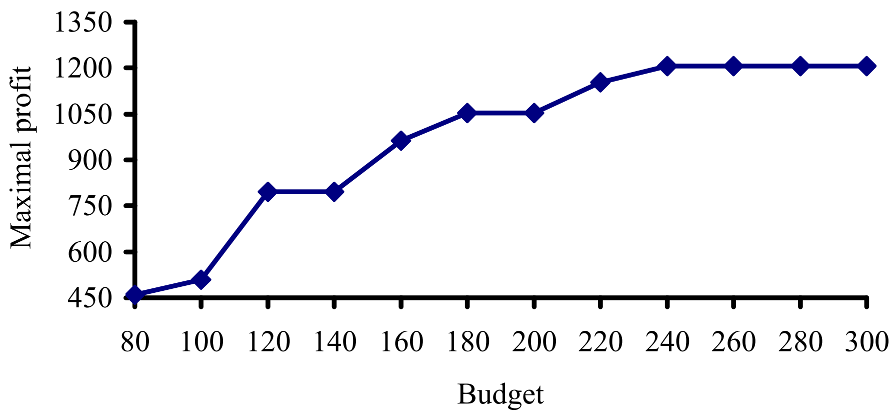


Figure 4.2 The maximal profit vs. the budget

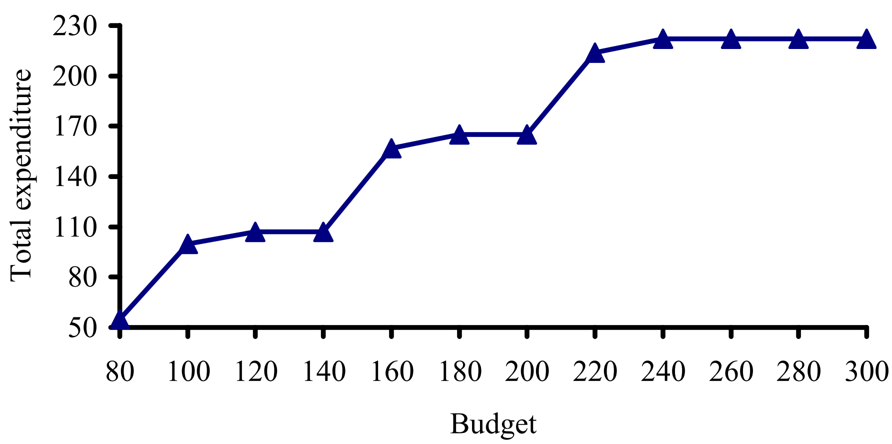


Figure 4.3 Total expenditure vs. budget

4.5.3 Examples for evaluating hybrid GA-LQP P-C method

Let us first use the above example with the first set of production capacities listed in Table 4.3 and budget $B = 200.0$ to evaluate performance of the hybrid GA-LQP P-C method for solving the MPEC model because the global optimal solution of the example has already been obtained. In the hybrid GA-LQP P-C method for solving this example, let the population size be $N = 500$, the penalty parameter involved in the fitness function expressed by eqn. (4.53) $\omega = 10.0$, and the maximal number of iterations be 15.

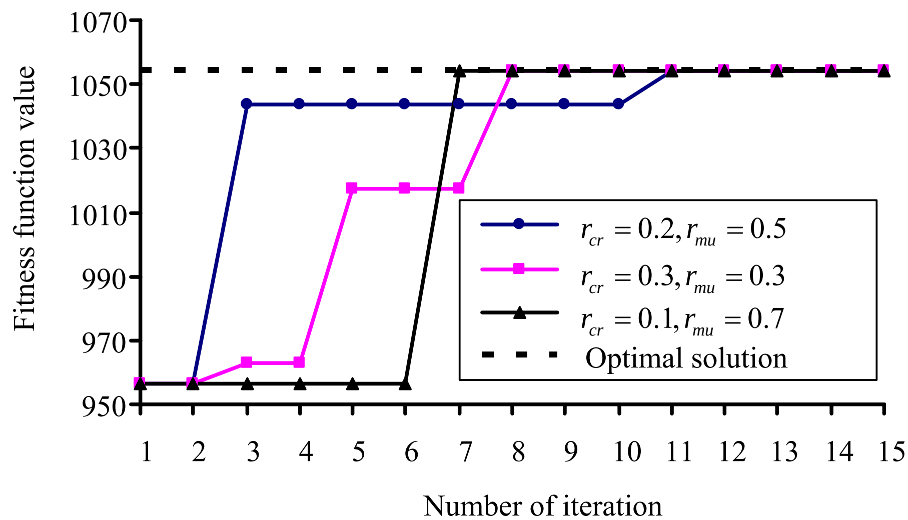


Figure 4.4 Change of the fitness function values of the small example solved by the hybrid GA-LQP P-C method

Figure 4.4 gives the change of the fitness function values obtained by the GA-LQP P-C method with three different combinations of the crossover and mutation probabilities. It fully shows that the hybrid GA-LQP P-C is able to find the optimal solution of the MPEC model of the example after ten iterations even if different sets of crossover and mutation probabilities are chosen.

Let us construct a relatively large competitive facility location problem with the maximization of profit with a budget constraint, for which the existing decentralized supply chain comprises 5 manufacturers, 25 retailers and 25 demand markets, namely,

$m = 5$, $n = 25$ and $o = 25$. The number of location candidates available to the entering firm is assumed to be 20. Production capacities and fixed setting up costs of facilities built in these twenty location candidates are listed in Table 4.5. The relevant production cost functions, handling cost functions, transaction cost functions and elastic demand functions take the same formulae shown by eqns. (4.54)-(4.58). It is still assumed that the budget $B = 200.0$.

Table 4.5 Production capacity and cost of a facility built at a location candidate for the large example

Location candidate	Production capacity	cost	Location candidate	Production capacity	cost
1	42	55	11	41	57
2	39	52	12	38	39
3	40	57	13	36	41
4	33	58	14	42	48
5	35	50	15	39	51
6	38	52	16	37	55
7	30	56	17	40	43
8	38	48	18	45	42
9	39	55	19	42	43
10	40	58	20	38	51

For the hybrid GA-LQP P-C method, let the population size $N = 500$, the crossover probability $r_{cr} = 0.2$, the mutation probability $r_{mu} = 0.5$, and the maximal number of iteration be equal to 100. The parameters used by the LQP P-C method are the same as that adopted by the numerical example in Subsection 5.2. Figure 4.5 depicts the stepwise increasing trend in terms of the fitness function value of the hybrid GA-LQP P-C method for solving this example. It indicates that the fitness function value reaches a maximum (i.e., the maximal profit) after seventy two iterations.

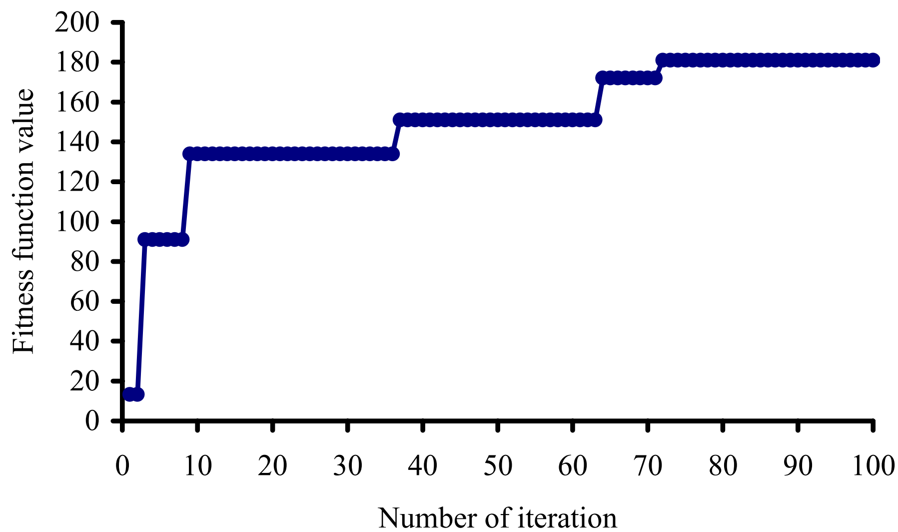


Figure 4.5 Change of the fitness function values of the large example solved by the hybrid GA-LQP P-C method

To conclude, it can be seen that the enumeration method incorporated with LQP P-C method can find the optimal solution for the small example. For the large example, the hybrid GA-LQP P-C method depicts its convergence.

4.6 Discussion and Summary

This chapter has proposed a new competitive facility location problem which significantly extends the competitive facility location problem with the SPE constraints. Since the limited production capacity of a manufacturer in a decentralized supply chain cannot be neglected, this chapter has successfully developed the SCNE model with production capacity constraints, which is formulated by the VI and solved by the LQP P-C method. By virtue of the SCNE model with the production capacity constraints, it has created an MPEC model for the proposed competitive facility location problem. Moreover, a meta-heuristics method called the hybrid GA-LQP P-C method has been designed for solving the MPEC model. The numerical examples have not only demonstrated the impact of production capacity constraints on the SCNE and the sensitivity analysis results of the MPEC model for the competitive facility location problem, but also shown the feasibility of the solution methods. To some extent, the proposed MPEC model in this chapter can more or less help a company to anticipate the reaction of the market after he enters the market. This can thus help the company to make decisions on locations of his plants.

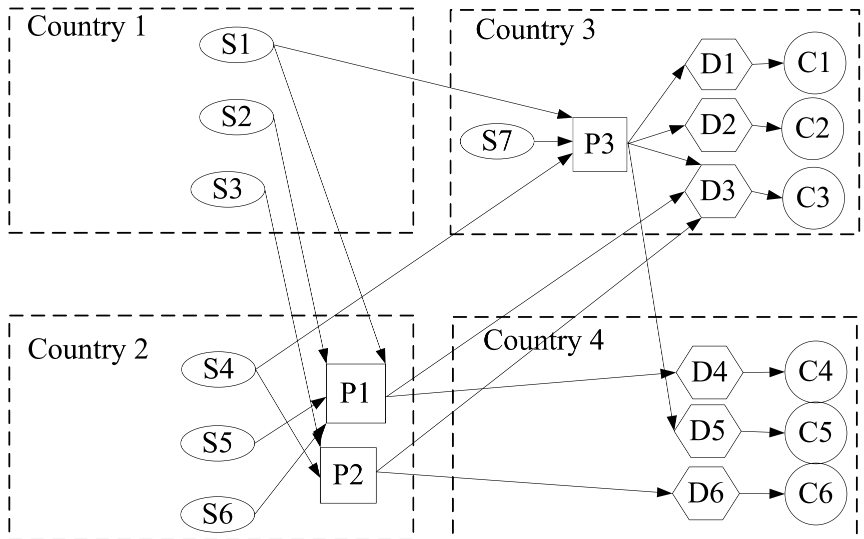
CHAPTER 5 MULTIPERIOD PRODUCTION-DISTRIBUTION PLANNING WITH TRANSFER PRICING AND DEMAND UNCERTAINTY

5.1 Introduction

With consideration of demand uncertainty, this chapter develops a chance constrained programming model for a global supply chain on a four-tier network comprising external raw materials suppliers, plants, DCs and customers. The objective of this model is to maximize the expected after-tax profit of an MNC over a taxation period by determining the number of product to be manufactured at each plant, the amount to be supplied to each DC, the optimal level of flows of raw materials from external vendors to plants, inventory levels of raw materials and products at each plant, and the transfer price between each plant and each DC. In order to capture the fluctuation of time-dependent parameters such as currency exchange rates, the taxation period is divided into several sub-periods. Assuming a stochastic demand in each sub-period, the model is formulated as a nondifferentiable maximization problem with chance constraints for inventory control strategy at each DC. To solve the model, a penalty function method embedded with a simulated annealing procedure will be carried out. After moving the chance constraints into the objective function to penalize the violation of chance constraints, the resulting model is a linear constrained maximization problem. Hence, Phrase I of the simplex method in linear program is employed to sample a solution in implementation of Metropolis Monte Carlo simulation in the simulated annealing procedure.

5.2 Problem Statement

Let us consider a global supply chain of an MNC. The supply chain consists of four tiers - external vendors of raw materials, plants, DCs and customers. The four tiers are located in different countries. It is assumed that the MNC owns all of the plants which produce a single product and all DCs which sell the product to the markets. The global supply chain operates as follows. Plants purchase raw materials from external vendors; in turn, DCs purchase the product from the plants and serve their own markets. For the sake of convenience, it is further assumed that each DC and its market are in the same country. Figure 5.1 provides a schematic example of a global supply chain network with seven external vendors located in three countries, three plants in countries two and three, six DCs in countries three and four and six markets served by these six DCs, respectively.



Legend:
S1-S7= External venders
P1-P3 = Plants
D1-D6= Distribution centers
C1-C6= Markets

Figure 5.1 A four-tier global supply chain network

Most countries have a fixed profit taxation period, typically one year, for MNCs. During the taxation period, a number of parameters including market demands and raw material price discounts offered by external vendors may fluctuate seasonally. To consider the impact of these time-dependent parameters on tactical decisions, the entire taxation period can be divided into several sub-periods.

Currency exchange rates are important issues in tactical supply chain decisions for an MNC. Normally it is difficult to forecast a currency exchange rate. Hence, in this chapter it is assumed that the currency exchange rates of each country in each sub-period are random.

Demand uncertainty is a practical concern for global supply chain managers because demand is forecasted when planning a global supply chain, and the forecast may be biased. Therefore, it is more reasonable to assume a stochastic demand faced by each DC in each sub-period rather than a deterministic demand. Inventory control at plants and DCs is vital to a global supply chain because inventory cost is a major component of the total cost. For example, in some sub-periods, plants may purchase more raw materials if the saving from price discounts is able to offset the additional inventory cost for one sub-period. With stochastic demand, the inventory at each DC becomes a random variable. Since each DC has limited inventory capacity, an inventory control policy is needed to ensure a level of confidence that the inventory will not exceed the capacity.

According to Organization for Economic Co-operation and Development (OECD) international guidelines based on the arm's length principle, the transfer price for a product are limited within a range around the real market price. As a requirement for the justification of transfer prices to tax authorities, all the transfer prices charged by the same plant must be the same (Vidal and Goetschalckx, 2001).

Finally, to make the calculations of cost and profit comparable, currencies of all the countries involved in the global supply chain are converted to a common currency, say, the US dollar.

Based on the preceding assumptions, the multiperiod supply chain planning with transfer pricing and demand uncertainty aims to maximize the expected value of after-tax profit of the MNC over the entire taxation period by determining the amount of production at each plant in each sub-period, the amount of supply to each DC in each sub-period, the optimal flow level of both raw materials from external vendors to plants in each sub-period, inventory levels of raw materials and products at each plant in each sub-period, and the transfer price charged by each plant in each sub-period.

5.3 Mathematical Model

Before presenting the model, the parameters and decision variables used in the model are stated. For the sake of representation, $inr_i^h(0)$, $inp_i(0)$ and $inv_j(0)$ are used to represent the inventory of raw materials in plants, final products in plants and the final products in DCs at the beginning of sub-period 1. Except for these, for clarity and ease of reading, all parameters are denoted by capital letters, and all decision variables are in lower case letters. Following are the parameters and decision variables.

Sets and indices

$[1, 2, \cdots, T]$	Set of sub-periods, indexed by *t*, and *T* is the total number of sub-periods.
H	Set of raw material types that are used to produce or assemble the product, indexed by *h*.
L	Set of countries where external vendors are located, indexed by *l*.
M	Set of countries where plants are located, indexed by *m*.
N	Set of countries where DCs are located, indexed by *n*.
P_m	Set of plants in country $m \in M$, indexed by i.
S_l^h	Set of vendors in country $l \in L$ that can supply raw material

type $h \in H$, indexed by s.

W_n Set of DCs in country $n \in N$, indexed by j.

Parameters associated with the international factors

$DUTY_{mn}(t)$ Import duty rate on value of the product shipped from country *m* to country *n* in sub-period t.

$E_m(t), E_n(t), E_l(t)$ Currency exchange rates of countries *m, n* and *l* to a common currency in sub-period t, respectively [dollar/monetary units of country *m, n* and *l*].

LTP_i, UTP_i Lower and upper bounds of transfer price of product charged by plant *i* to any DC [monetary units of the country where plant i is located /unit of the product].

TAX_m, TAX_n, TAX_l Corporate profit tax rates in countries *m*, *n* and *l*, respectively.

Parameters associated with costs, prices and capacities

$SCR_s^h(t)$ Supply capacity of raw material type h of vendor s in sub-period *t* [number of units of raw material type *h*].

$PCAP_i$ Production capacity of plant i during one sub-period [number of units of products]

ICR_i^h Inventory capacity of raw material type h at plant i [number of units of raw material type *h*].

ICP_i Inventory capacity of the product at plant i [number of units of the product].

ICW_i Inventory capacity of the product at DC j [number of units of the product].

TC_{ij} — Transaction cost per unit (not including the import duties) of the product shipped from plant i to DC j [monetary units of country where plant i is located/unit of the product].

PC_i — Production cost per unit of the product at plant i [monetary units of country where plant i is located/unit of the product].

SP_s^h — Selling price of raw material h by vendor s [monetary units of country where vendor s is located/ unit of raw material type h].

TC_{si}^h — Transaction cost per unit of raw material type h obtained by plant i from raw material supplier s [monetary units of country where plant i is located / unit of raw material h].

IC_i^h — Inventory cost per unit of raw material type h in plant i for one sub-period [monetary units of country where plant i is located / unit of raw material h per sub-period].

IC_i — Inventory cost per unit of the product at plant i for one sub-period [monetary units of country where plant i is located / unit of the product per sub-period].

MP_j — Market price of the product at DC j [monetary units of country of DC j].

IC_j^1 — Inventory cost per unit of the product at DC j for one sub-period [monetary units of country where DC j is located /

unit of the product per sub-period].

IC_j^2 Outsourced inventory cost per unit of product at DC j for one sub-period in the case that the DC's warehouse cannot accommodate all the product [monetary units of country where DC j is located / unit of the product per sub-period].

Parameters associated with stochastic demand, bill of material and raw material discount rates

AMO^h Quantity of raw material of type h needed to produce one unit of the product [number of units of raw material type h / unit of the product].

$D_j(t)$ Stochastic demand for the product at DC j in sub-period t [number of unit of the product].

$f_j(t,x)$ Probability density function of random variable $D_j(t)$, where x is the random variable.

$DIS_{si}^h(t)$ Price discount rate of raw material type h supplied by vendor s to plant i in sub-period t.

Miscellaneous parameters

$inr_i^h(0)$ Inventory of raw material type h at plant i at the beginning of sub-period 1 [number of units of raw material type h].

$inp_i(0)$ Inventory of the product at plant i at the beginning of sub-period 1 [number of units of the product].

$inv_j(0)$ Inventory of the product at DC j at the beginning of sub-period 1 [number of units of the product]

Decision variables

$tp_i(t)$ Transfer price of the product, charged by plant i to any DC in sub-period t [monetary units of country where plant i is located/unit of the product].

$pp_i(t)$ Number of units of product produced by plant i in sub-period t [number of units of the product].

$pw_{ij}(t)$ Quantity of the products, produced at plant i and shipped to DC j in sub-period t [number of units of the product].

$raw_{si}^h(t)$ Quantity of raw material type h supplied by vendor s to plant i in sub-period t [number of units of raw material h].

$inr_i^h(t)$ Inventory of raw material h in plant i during sub-period t [number of units of raw material h].

$inp_i(t)$ Inventory of the product in plant i during sub-period t [number of units of the product]

For the sake of presentation, let $X(t)$ denote the vector of all the decision variables associated with sub-period t, $t = 1, 2, \cdots, T$, and X be the vector of all the decision variables, namely,

$$X(t) = \begin{bmatrix} tp_i(t), pp_i(t), pw_{ij}(t), raw_{si}^h(t), inr_i^h(t), inp_i(t): \\ i \in P_m, j \in W_n, s \in S_l^h, m \in M, n \in N, l \in L, h \in H \end{bmatrix}, t = 1, 2, ..., T \tag{5.1}$$

$$X = \left[X(t) : t = 1, 2, \cdots, T \right] \tag{5.2}$$

5.3.1 Expected value of after-tax profit for a plant

In sub-period t, the profit of plant i in country $m \in M$ is defined by the revenue from selling the product to all DCs minus production cost, transaction cost, inventory cost and the cost for raw materials purchasing. Since the currency exchange rates $E_m(t)$ and $E_l(t)$ are random, the expected value of the profit of plant i in sub-period t, denoted by $EPP_i(X(t))$ is expressed as:

$$\begin{aligned} EPP_i\left(X(t)\right) = E\Bigg(& E_m(t) \sum_{n \in N} \sum_{j \in W_n} \left[pw_{ij}(t) \times \left(tp_i(t) - TC_{ij} \right) \right] \\ & - E_m(t) \left[pp_i(t) \times PC_i + inp_i(t) \times IC_i \right] - E_m(t) \sum_{h \in H} \left[inr_i^h(t) \times IC_i^h \right] \\ & - \sum_{h \in H} \sum_{l \in L} \sum_{s \in S_l^h} \left[E_m(t) \times raw_{si}^h(t) \times TC_{si}^h + E_l(t) \times raw_{si}^h(t) \times SP_s^h \times DIS_{si}^h(t) \right] \Bigg) \\ & , i \in P_m, m \in M \end{aligned} \tag{5.3}$$

The right hand side of equation (5.3) comprises of four terms. The first term is the revenue from selling the product to all the DCs minus the transaction cost between the plant and the DCs; the second term is the sum of production and inventory costs of the product in sub-period t; the third term is the inventory cost of raw materials in sub-period t; the fourth term is the sum of transaction cost between the plant and external raw material suppliers and the expense of purchasing the raw materials.

Adding up the profit shown in equation (5.3) over all sub-periods in the entire taxation duration yields the expected before-tax profit made by plant i, which is a function of all the decision variables X, denoted by $EBTPP_i(X)$:

$$EBTPP_i(X) = \sum_{i=1}^{T} EPP_i\left(X(t)\right), i \in P_m, m \in M \tag{5.4}$$

Because tax is levied only if a plant makes a profit over the entire taxation period, the after-tax profit made by plant i, denoted by $EATPP_i(X)$, can be expressed by a step-wise function:

$$EATPP_i(X) = \begin{cases} (1 - TAX_m) BTPP_i(X), \text{ if } EBTPP_i(X) > 0 \\ BTPP_i(X), \text{ otherwise} \end{cases}, i \in P_m, m \in M \quad (5.5)$$

5.3.2 Expected value of after-tax profit for a DC

Demand uncertainty of the product at DC j in sub-period t causes stochastic inventory, which is denoted by $inv_j(t)$:

$$inv_j(t) = \left[\sum_{m \in M} \sum_{i \in P_m} pw_{ij}(t) + inv_j(t-1) - D_j(t) \right]^+ \quad (5.6)$$

where

$$\left[\sum_{m \in M} \sum_{i \in P_m} pw_{ij}(t) + inv_j(t-1) - D_j(t) \right]^+ = \max\left(\sum_{m \in M} \sum_{i \in P_m} pw_{ij}(t) + inv_j(t-1) - D_j(t), 0 \right) .$$

The term $\sum_{m \in M} \sum_{i \in P_m} pw_{ij}(t)$ in the right hand side of equation (5.6) is the number of products purchased by DC j from all the plants in sub-period t; the term $inv_j(t-1)$ is the inventory of the product in the sub-period $(t-1)$. Note that since the demand faced by DC j in sub-period t, $D_j(t)$, is a random variable, $inv_j(t)$ is also a random variable. The probability density function of $inv_j(t)$ can be obtained in subsection 5.3.3. In reality, equation (5.6) indicates a fundamental phenomenon that inventory exists if and only if the supply exceeds demand. In addition, it should be pointed out that as $inv_j(t)$ exceeds the inventory capacity of DC j, the DC has to look for a thirty-party warehouse to store the inventory over capacity. It results in a higher unit inventory cost than the inventory cost at DC j. Hence, the inventory cost of DC j during sub-period t, denoted by $TIC_j(t)$, is expressed as:

$$TIC_j(t) = IC_j^1 \times inv_j(t) + IC_j^2 \times \left[inv_j(t) - ICW_j\right]^+ \tag{5.7}$$

where $\left[inv_j(t) - ICW_j\right]^+ = \max\left(inv_j(t) - ICW_j, 0\right)$. The first term of right hand side of equation (5.7) represents the cost of storing products in DC j, while the second term represents the cost of storing products in a thirty party warehouse.

In sub-period t, the real quantity of the product from the DC j purchased by customers is a random variable, denoted by $rd_j(t)$, is expressed as follows:

$$rd_j(t) = \min\left(D_j(t), \sum_{m \in M} \sum_{i \in P_m} pw_{ij}(t) + inv_j(t-1)\right) \tag{5.8}$$

Equation (5.8) describes a fact that the number of the products purchased by consumers cannot exceed the available supply of the product. Thus, the revenue of DC j from selling the product in sub-period t is a random variable defined by

$$TIW_j(t) = MP_j \times rd_j(t) \tag{5.9}$$

It is straightforward to check that

$$\begin{aligned} rd_j(t) &= D_j(t) + \min\left(0, \sum_{m \in M} \sum_{i \in P_m} pw_{ij}(t) + inv_j(t-1) - D_j(t)\right) \\ &= \sum_{m \in M} \sum_{i \in P_m} pw_{ij}(t) + inv_j(t-1) - \max\left(0, \sum_{m \in M} \sum_{i \in P_m} pw_{ij}(t) + inv_j(t-1) - D_j(t)\right) \\ &= \sum_{m \in M} \sum_{i \in P_m} pw_{ij}(t) + inv_j(t-1) - inv_j(t), t = 1, 2, \cdots, T, j \in W_n, n \in N \end{aligned} \tag{5.10}$$

According to equations (5.9)-(5.10), the revenue obtained from selling products for DC j in sub-period t, denoted by $TIW_j(t)$, can be calculated by

$$TIW_j(t) = MP_j \times rd_j(t) = MP_j \times \left(\sum_{m \in M} \sum_{i \in P_m} pw_{ij}(t) + inv_j(t-1) - inv_j(t)\right) \tag{5.11}$$

The expected profit in sub-period t for DC j, denoted by $EPW_j(X, t)$ can be expressed as follows:

$$\begin{aligned} EPW_j(X,t) = E\Big(& E_n(t) \times TIW_j(t) - E_n(t) \times TIC_j(t) \\ & - \sum_{m \in M} \sum_{i \in M} \left[E_m(t) \times pw_{ij}(t) \times tp_i(t) \times \left(1 + DUTY_{mn}(t)\right) \right] \Big) \end{aligned} \tag{5.12}$$

There are three terms in the right hand side of equation (5.12). The first term is the revenue from selling the product to consumers; the second term is the inventory cost and the third term is for the expense of purchasing the product from the plants, including import duties. Accordingly, the expected before-tax profit gained by a DC over the entire taxation period can be calculated as follows:

$$EBTPW_j(X) = \sum_{t=1}^{T} EPW_j(X,t), \ j \in W_n, n \in N \tag{5.13}$$

The expected value of the after-tax profit for DC j, denoted by $EATPW_j(X)$, can be expressed by the step-wise function:

$$EATPW_j(X) = \begin{cases} (1 - TAX_n) EBTPW_j(X), & \text{if } EBTPW_j(X) > 0 \\ EBTPW_j(X), & \text{otherwise} \end{cases}, \ j \in W_n, n \in N \tag{5.14}$$

5.3.3 Probability density function of inventory for final products in each DC

Let us first derive the probability density function of random variable $inv_j(1)$. According to equation (5.6), it follows that

$$inv_j(1) = \max\left(0, \sum_{m \in M} \sum_{i \in P_m} pw_{ij}(1) + inv_j(0) - D_j(1)\right) \tag{5.15}$$

For any $x > 0$, cumulative distribution function of random variable $inv_j(1)$, denoted by $F_j^I(1,x)$, can be expressed as follows:

$$\begin{aligned} F_j^I(1,x) &= P\left[\sum_{m \in M} \sum_{i \in P_m} pw_{ij}(1) + inv_j(0) - D_j(1) \le x \right] \\ &= 1 - F_j\left(1, \sum_{m \in M} \sum_{i \in P_m} pw_{ij}(1) + inv_j(0) - x\right) \end{aligned} \tag{5.16}$$

where $F_j(1,x)$ is the cumulative distribution function of stochastic demand $D_j(1)$.

Let $f_j^I(1,x)$ denote the probability density function of $inv_j(1)$, and equation (5.16) implies that

$$f_j^I(1,x) = \frac{dF_j^I(1,x)}{dx} = f_j\left(\sum_{m \in M}\sum_{i \in P_m} pw_{ij}(1) + inv_j(0) - x\right) \tag{5.17}$$

where $0 < x \le \sum_{m \in M}\sum_{i \in P_m} pw_{ij}(1) + inv_j(0)$.

In addition, the probability of $inv_j(1) = 0$, denoted by $f_j^I(1,0)$, can be expressed as follows:

$$\begin{aligned} f_j^I(1,0) &= P\left(inv_j(t-1) = 0\right) = P\left(\max\left(0, \sum_{m \in M}\sum_{i \in P_m} pw_{ij}(1) + inv_j(0) - D_j(1)\right) = 0\right) \\ &= P\left(\sum_{m \in M}\sum_{i \in P_m} pw_{ij}(1) + inv_j(0) - D_j(1) \le 0\right) \\ &= 1 - F_j\left(1, \sum_{m \in M}\sum_{i \in P_m} pw_{ij}(1) + inv_j(0)\right) \end{aligned} \tag{5.18}$$

For $x > \sum_{m \in M}\sum_{i \in P_m} pw_{ij}(1) + inv_j(0)$, the probability density function of $inv_j(1)$ is

$$f_j^I(1,0) = 0 \tag{5.19}$$

It is now ready to derive the probability density function of random variable $inv_j(t)$ for $t \ge 2$. Let $F_j^I(t,x)$ denote the cumulative distribution function of random variable $inv_j(t)$. Eqn (5.6) implies that

$$
\begin{aligned}
F_j^I(t,x) &= P\left(\max\left(\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(t) + inv_j(t-1) - D_j(t), 0 \right) \le x \right) \\
&= P\left(\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(t) + inv_j(t-1) - D_j(t) < x \,\middle|\, inv_j(t-1) = 0 \right) \times P\left(inv_j(t-1)=0\right) + \\
&\int_0^{\sum_{\tau=1}^{t-1}\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(\tau)+inv_j(0)} P\left(\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(t) + inv_j(t-1) - D_j(t) < x \,\middle|\, inv_j(t-1) = \omega \right) \times f_j^I(t-1,\omega)d\omega \\
&\left(1 - F_j\left(\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(t) - x \right)\right) \times f_j^I(t-1,0) + \\
&\int_0^{\sum_{\tau=1}^{t-1}\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(\tau)+inv_j(0)} \left(1 - F_j\left(\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(t) + \omega - x \right)\right) \times f_j^I(t-1,\omega)d\omega
\end{aligned}
$$

(5.20)

Thus, for $0 < x \le \sum_{\tau=1}^{t}\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(\tau) + inv_j(0)$, the probability density function of $inv_j(t)$, denoted by $f_j^I(t,x)$, can be expressed as:

$$
\begin{aligned}
f_j^I(t,x) = f_j&\left(\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(t) - x \right) \times f_j^I(t-1,0) + \\
&\int_0^{\sum_{\tau=1}^{t-1}\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(\tau)+inv_j(0)} f_j\left(\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(t) + \omega - x \right) \times f_j^I(t-1,\omega)d\omega
\end{aligned}
\tag{5.21}
$$

The probability of $inv_j(t) = 0$ can be expressed as:

$$
\begin{aligned}
f_j^I(t,x) = &\left(1 - F_j\left(\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(t) \right)\right) \times f_j^I(t-1,0) + \\
&\int_0^{\sum_{\tau=1}^{t-1}\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(\tau)+inv_j(0)} \left(1 - F_j\left(\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(t) + \omega \right)\right) \times f_j^I(t-1,\omega)d\omega
\end{aligned}
\tag{5.22}
$$

For $x > \sum_{\tau=1}^{t}\sum_{m\in M}\sum_{i\in P_m} pw_{ij}(\tau) + inv_j(0)$, the probability density function of $inv_j(t)$ is:

$$
f_j^I(t,x) = 0 \tag{5.23}
$$

5.3.4 Chance constrained programming model

Expected value of the after-tax profit for the MNC, denoted by function $ATP(X)$, is equal to the sum of after-tax profit for all the plants shown in eqn. (5.5) and the expected value of the after-tax profit for all the DCs shown in eqn. (5.14), namely,

$$ATP(X) = \sum_{m \in M} \sum_{i \in P_m} EATPP_i(X) + \sum_{n \in N} \sum_{j \in P_n} EATPW_j(X) \tag{5.24}$$

The multiperiod global supply chain planning with transfer pricing and demand uncertainty can be formulated into the following maximization model:

$$\text{Maximize } ATP(X) \tag{5.25}$$

s.t.

$$inr_i^h(t) = inr_i^h(t-1) + \sum_{l \in L} \sum_{s \in S_l^h} raw_{si}^h(t) - AMO^h \times pp_i(t), i \in P_m, m \in M, h \in H, t = 1, ..., T \tag{5.26}$$

$$inp_i(t) = inp_i(t-1) + pp_i(t) - \sum_{n \in N} \sum_{j \in W_n} pw_{ij}(t), i \in P_m, m \in M, t = 1, 2, \cdots, T \tag{5.27}$$

$$\sum_{m \in M} \sum_{i \in P_m} raw_{si}^h(t) \le SCR_s^h(t), s \in S_l^h, h \in H, l \in L, t = 1, 2, \cdots, T \tag{5.28}$$

$$inp_i(t) \le ICP_i, i \in P_m, m \in M, t = 1, 2, \cdots, T \tag{5.29}$$

$$inr_i^h(t) \le ICR_i^h, i \in P_m, m \in M, t = 1, 2, \cdots, T \tag{5.30}$$

$$\Pr\left(inv_j(t) \le ICW_j\right) \ge 1 - \alpha_j, j \in W_n, n \in N, t = 1, 2, \cdots, T \tag{5.31}$$

$$LTP_i \le tp_i(t) \le UTP_i, i \in P_m, m \in M \tag{5.32}$$

$$pp_i(t) \le PCAP_i, i \in P_m, m \in M, t = 1, 2, ..., T \tag{5.33}$$

$$X(t) \ge 0, t = 1, 2, \cdots, T \tag{5.34}$$

Eqn. (5.25) is the objective function which is to maximize the expected valve of the after-tax profit. Eqns. (5.26)-(5.27) are inventory conservation equations for all raw materials and the product at a plant in each sub-period, respectively. Note that parameters, $AMO^h, h \in H$ in eqn. (5.26) reflect the bill of materials. Eqn. (5.28) is the supply capacity constraint of each type of raw material at a vendor in each sub-period. Eqns. (5.29)-(5.30) are inventory capacity constraints of a raw material type and the product at plant i in each sub-period, respectively. Eqn. (5.31) is the chance constraint ensuring that the probability of inventory of the product at DC j in each sub-period greater than the inventory capacity is at most α_j, where α_j is a pre-specified threshold. Eqn. (5.32) gives the feasible interval of transfer price. Eqn. (5.33) is the production capacity constraint for each plant. Eqn. (5.34) represents nonnegativity of the decision variables.

Because the after-tax profits gained by all the plants and DCs are step-wise functions with respect to the decision variables, the objective function defined by eqn. (5.24) is a nondifferentiable function. Therefore, the chance constrained programming model (5.25)-(5.34) is a nondifferentiable optimization problem.

5.4 Solution Algorithm

As shown in subsection 5.3.3, the probability density function of $inv_j(t)$ depends on the probability density function of $inv_i(t-1)$ and the decision variables, $pw_{ij}(1)$, $pw_{ij}(2)$, $\cdots$, $pw_{ij}(t)$ for all $i \in P_m, m \in M$. The chance constraints represented by eqn. (5.31), thus, are nonlinear and nonconvex with respect to decision variable X. Although it is able to convert the nondifferentiable objective function (5.25) into a continuously differentiable one by the technique used by Vidal and Goetschalckx (2001), the resulting model is still a nonconcave maximization problem that is one of intractable problems with classical algorithms in nonlinear programming. Alternatively, simulated annealing (SA) method, one of the artificial intelligent algorithms, can be applied to solve the chance constrained programming model (5.25)

-(5.34) since SA method has been successfully applied to some intractable optimization models arising from production planning and control (Onwubolu, 2002).

The simulated annealing method originated from an analogy with the physical annealing process to find low energy states of a solid in a heat bath (Metropolist et al., 1953). It is a stochastic method to avoid getting stuck in a local, non-global optimum in a search for the global optimum. This is made by accepting transitions corresponding to a decrease in function value in addition to transitions corresponding to an increase in function value. The latter is done in a limited way by means of a stochastic acceptance criterion. In the course of the maximization process, probability of accepting deteriorations descends slowly towards zero by using a cooling schedule. These 'deteriorations' make it possible to climb out of the local optimum and explore the feasible region of the problem entirely. This procedure will lead to a (near) global optimum. In general, an SA method consists of two main steps - Metropolis Monte Carlo Simulation and cooling schedule (Kirkpatrick et al., 1983).

The chance constraints in model (5.25)-(5.34) bring a computational challenge in implementation of the Metropolis Monte Carlo Simulation due to their nonlinearity and nonconvexity. To overcome such a difficulty, these chance constraints are put into the objective function (5.25) by means of the penalty function method (Bazaraa et al., 1993). The induced linearly constrained optimization model with a penalty function can be formulated as follows:

$$\underset{X \in \Omega}{\text{Maximize}}\ \Psi\left(X,\mu\right) = ATP\left(X\right) - \mu \sum_{t=1}^{T} \sum_{n \in N} \sum_{j \in W_n} \max\left(Z_{\alpha_j}^{inv_j(t)}\left(X\right) - ICW_j, 0\right) \quad (5.35)$$

where μ is a positive penalty parameter, Ω is the set of feasible solutions satisfying constraints (5.26)-(5.30) and (5.32)-(5.34), namely,

$$\Omega = \left\{ X \middle| X \text{ satisfies constraints (26)-(30) and (32)-(34)} \right\} \tag{5.36}$$

$Z_{\alpha_j}^{inv_j(t)}(X)$ is a number such that the probability of random variable $inv_j(t)$ greater than this number is equal to α_j for a given $X \in \Omega$, i.e.,

$$\Pr\left(inv(t) \geq Z_{\alpha_j}^{inv_j(t)}(X)\right) = \alpha_j \tag{5.37}$$

According to the probability definition, it is easy to verify that

$$\Pr\left(inv_j(t) \leq ICW_j\right) \geq 1-\alpha_j \quad \text{if and only if } Z_{\alpha_j}^{inv_j(t)}(X) \leq ICW_j, j \in W_n, n \in N \tag{5.38}$$

Therefore, the second term in the right hand side of eqn. (5.35) penalizes violation of the chance constraints in the model (5.25)-(5.34), and it is referred to as the penalty function.

It can be seen that all the constraints defining set Ω are linear. Hence, the following SA method can be employed to obtain a solution to the linearly constrained maximization model (5.35) beginning with an initial solution $X^{(0)} \in \Omega$.

Simulation annealing procedure for solving the linearly constrained maximization model (5.35)

Step 0: (Initialization) Let initial solution $X^{(0)} \in \Omega$, $\tau^{(0)}$ and $\hat{\tau}$ be the initial and final temperatures, $0 < \sigma < 1$ be the parameter in the temperature cooling schedule, $IteMax$ be the maximum number of iterations used in the Metropolis Monte Carlo Simulation. Let UB be a predetermined vector with the same dimension of the decision variable vector X, $k := 0$ and $k' := 0$.

Step 1: (Metropolis Monte Carlo Simulation)

Step 1.1: (Sampling) Randomly generate a vector $\xi \in (0, UB]$ and then gain a solution in set $\Omega \cap \left\{ X^{(k)} - \xi \le X \le X^{(k)} + \xi \right\}$, denoted by $\overline{X}^{(k)}$, by applying Phase I of the simplex method in linear program, where $UB = \max\left(1, X^{(k)}/100\right)$.

Step 1.2: (Boltzmaann acceptance criterion) If $\Psi\left(\overline{X}^{(k)}\right) > \Psi\left(X^{(k)}\right)$, then let $X^{(k+1)} = \overline{X}^{(k)}$. Otherwise, randomly generate a number $\lambda_1 \in (0,1)$ and perform the operation:

$$X^{(k+1)} = \begin{cases} \overline{X}^{(k)}, & \text{if } \lambda_1 \le \exp\left\{\left[\Psi\left(\overline{X}^{(k)}\right) - \Psi\left(X^{(k)}\right)\right] / \tau^{(k')}\right\} \\ X^{(k)}, & \text{otherwise} \end{cases} \tag{5.39}$$

Step 1.3: (Sampling termination criterion) Let $k := k+1$. If $k = IteMax$, go to Step 2; otherwise, go to Step 1.1.

Step 2: (Perform a proportional cooling schedule) Let $\tau^{(k'+1)} := \sigma\tau^{(k')}$ and $k' := k'+1$, go to Step 3.

Step 3: (Stop criterion) If $\tau^{(k')} > \hat{\tau}$, let $X^{(0)} = X^{(k)}$ and $k := 0$, go to Step 1. Otherwise, stop and the output is $X^{(k)}$

The above SA procedure starts with the Metropolis Monte Carlo simulation at a high temperature and consists of a pair of nested Do-loops. The outer loop sets the temperature and the inner-most loop, namely, Step 1, runs a Metropolis Monte Carlo simulation at that temperature. Step 2 presents a proportional cooling schedule to decrease the temperature. As set Ω is a polyhedral, compared to the interval constraints, it is not easy to sample a feasible solution. Thus, there is a need of a systematic approach that can randomly generate a solution in a polyhedral. Step 1.1 contributes such an approach by employing Phrase I of the simplex method in linear program to sample in the set of feasible solutions. Step 1.2 is the typical Boltzmann acceptance criterion that is able to accept a solution with a lower objective function value.

Having a solution to the linearly constrained maximization model (5.35) corresponding to penalty parameter μ, the penalty function method will increase the value of the penalty parameter if the solution violates any chance constraints. The penalty function method embedded with the simulated annealing procedure can be presented below.

Penalty function method embedded with the simulated annealing procedure

Step 0: (Initialization) Let $\varepsilon > 0$ be a termination tolerance, μ_0 be an initial positive penalty parameter and a known parameter $\beta > 1$. Let $K = 1$ and $\mu = \mu_0$.

Step 1: (Find an initial solution) Find a feasible solution in set Ω, denoted by $Y^{(K)}$, by Phrase I of the simplex method.

Step 2: (Invoke the SA procedure) Figure a solution to the maximization model (5.35) associated with penalty parameter μ, denoted by $Y^{(K+1)}$, by the preceding simulated annealing procedure starting with $X^{(0)} = Y^{(K)}$.

Step 3: (Check a stop criterion) If $\mu \sum_{t=1}^{T} \sum_{n \in N} \sum_{j \in W_n} \max\left(Z_{\alpha_j}^{inv_j(t)} \left(Y^{(k+1)} \right) - ICW_j, 0 \right) \leq \varepsilon$, then stop and output $Y^{(K+1)}$; otherwise, go so Step 4.

Step 4: (Enlarge the penalty parameter) Let $\mu := \beta\mu$, $K := K + 1$ and go to Step 2.

Convergence of the above method can be guaranteed if the simulated annealing method can lead to a global optimum of linearly constrained maximization (5.35) at Step 2 (see, Bazaraa et al., 1993). Although the simulated annealing procedure is able to theoretically find a global optimum for an optimization problem, however, it usually terminates at a local optimal solution in a limited computational time. In other words, the penalty function method embedded with the simulated annealing procedure is a heuristic algorithm.

5.5 Numerical Examples

In this section, hypothetical examples are constructed to demonstrate the model and solution algorithm since it is difficult to obtain any example in the literature with sufficient data for this problem. Assume that an MNC assembling personal computers (PCs) possesses a global supply chain comprising five external vendors, three plants and five DCs serving five markets as shown in Figure 5.2. These three assembling plants are hypothetically assumed to be located in Thailand, Mexico and India, respectively, and they purchase CPUs, mainboards, hard disks, DVD-ROMS and monitors, forming five types of raw materials, from five external vendors located in Taiwan. It is further assumed that these five DCs are located in USA, Britain, Canada, Germany and Japan, respectively, and that each vendor is able to provide all five parts (raw materials). The objective of the company is to maximize its overall after-tax profit over a one-year taxation period, which is divided into four quarters, namely, $T = 4$. As for the demand uncertainty, it is assumed that the stochastic demand in each quarter in each market follows a normal distribution.

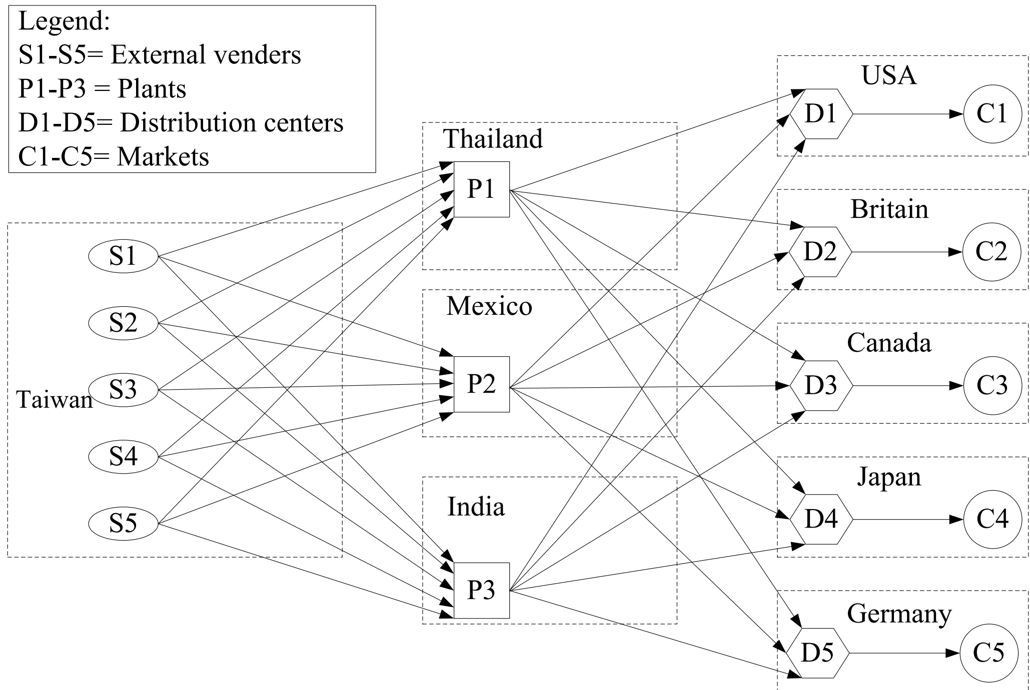


Figure 5.2 Global supply chain network of the numerical example

Market price, time-dependent discount and supply capacity for each type of raw material are tabulated in Tables 5.1-5.3. Tables 5.4-5.5 list the unit transaction and inventory costs of each type of material at a plant; Tables 5.6-5.7 give the unit production and inventory costs of PCs at each plant; Tables 5.8-5.9 present production and inventory capacities of PCs at each plant; Tables 5.10-5.11 show inventory capacity of each type of raw material at each plant and bill of the parts (materials) to produce a PC. Regarding each DC, the unit transaction cost between a plant and the DC, the unit inventory and outsourced inventory costs of PCs, and the inventory capacity of PCs are shown in Tables 5.12-5.15. As to the international factors, time-dependent currency exchanges rates, profit tax rate in each country and allowable intervals of transfer pricing are shown in Tables 5.16-5.18, respectively. Table 5.19 gives market price of PCs in each market; Tables 5.20-5.21 list mean and standard derivation of the normal distribution for the stochastic demand in each sub-period in a market, respectively.

Table 5.1 Prices of raw materials

Raw material	CPU	Hard disk	Mainboard	DVD-ROM	Monitor
Price (TWD/Unit)	4200	3000	4500	1200	8400

Table 5.2 Discount of each type of raw material in each sub-period

Vendor	Sub-Period	1	2	3	4
1	CPU	0.92	1	1	0.9
	Hard disk	1	0.94	0.92	0.95
	Mainboard	0.9	0.94	0.92	0.95
	DVD-ROM	0.99	0.93	0.96	0.91
	Monitor	1	0.96	0.94	0.95
2	CPU	0.9	0.94	0.88	0.95
	Hard disk	0.97	0.95	0.92	0.97
	Mainboard	0.92	0.93	0.93	1
	DVD-ROM	0.94	0.98	0.92	0.96
	Monitor	0.97	1	0.86	0.97

Table 5.2 Discount of each type of raw material in each sub-period (Continued)

3	CPU	0.99	0.96	0.93	0.95
	Hard disk	0.93	0.98	1	0.94
	Mainboard	0.95	1	0.87	0.93
	DVD-ROM	0.96	0.93	0.8	0.97
	Monitor	0.94	0.9	0.84	0.96
4	CPU	0.99	0.93	0.7	0.93
	Hard disk	0.94	0.92	0.93	0.95
	Mainboard	0.96	0.88	0.85	1
	DVD-ROM	0.95	0.97	0.92	0.93
	Monitor	0.98	0.86	0.94	0.97
5	CPU	1	0.88	0.94	0.99
	Hard disk	0.97	0.92	0.93	0.94
	Mainboard	1	0.89	1	0.95
	DVD-ROM	0.94	0.94	0.85	0.96
	Monitor	0.95	0.96	0.88	1

Table 5.3 Supply capacity of raw materials of each vendor in each sub-period (Unit)

Vendor	Sub-period	1	2	3	4
1	CPU	300	280	290	350
	Hard disk	150	140	145	180
	Mainboard	190	180	190	210
	DVD-ROM	210	200	205	240
	Monitor	220	200	210	220

Table 5.3 Supply capacity of raw materials of each vendor in each sub-period (Unit) (Continued)

2	CPU	150	160	180	140
	Hard disk	200	210	220	160
	Mainboard	230	240	250	200
	DVD-ROM	210	220	230	180
	Monitor	220	240	250	190
3	CPU	220	200	190	240
	Hard disk	210	195	180	230
	Mainboard	220	190	185	225
	DVD-ROM	150	140	135	180
	Monitor	180	170	160	170
4	CPU	210	220	160	130
	Hard disk	190	200	150	120
	Mainboard	195	200	140	110
	DVD-ROM	190	210	160	140
	Monitor	180	190	145	130
5	CPU	200	190	140	200
	Hard disk	200	190	130	160
	Mainboard	240	220	160	180
	DVD-ROM	210	200	150	180
	Monitor	180	170	110	175

Table 5.4 Unit transaction cost related to raw materials at each plant (TWD/Unit)

Plant	CPU	Hard disk	Mainboard	DVD-ROM	Monitor
1(Thailand)	300	200	320	80	600
2(Mexico)	600	410	640	160	1200
3(India)	350	225	360	90	700

Table 5.5 Unit inventory cost of each type of raw material at each plant

Raw Material	CPU	Hard disk	Mainboard	DVD-ROM	Monitor
Plant 1(THB/Unit/Unit time)	20	19	21	12.5	400
Plant 2(MXN/Unit/Unit time)	5	4.5	5.25	3	9.5
Plant 3(INR/Unit/Unit time)	19.5	19	20.5	12	39

Table 5.6 Unit assembly cost of PCs at each plant

Plant	1(THB/Unit)	2(MXN/Unit)	3(INR/Unit)
Assembly cost	140	30	150

Table 5.7 Unit inventory cost of PCs at each plant

DC	Thailand(THB/unit)	Mexico(MXN/unit)	India(INR/unit)
Inventory cost	500	150	520

Table 5.8 Production capacity of each plant

Plant	1	2	3
Capacity (Unit)	200	250	230

Table 5.9 Inventory capacity of PCs at each plant

Plant	1	2	3
Capacity (Unit)	300	350	320

Table 5.10 Inventory capacity of each type of raw material at each plant

Raw Material	CPU	Hard disk	Mainboard	DVD-ROM	Monitor
Plant 1(Unit)	500	300	450	400	300
Plant 2(Unit)	450	250	430	350	300
Plant 3(Unit)	400	200	410	340	250

Table 5.11 Bill of material

Raw material	CPU	Hard disk	Mainboard	DVD-ROM	Moritor
BOM	1	1	1	1	1

Table 5.12 Unit transaction cost between each plant and each DC

DC	1 (USA)	2 (Britain)	3 (Canada)	4 (Germany)	5 (Japan)
Plant 1 (THB/Unit)	50	45	55	43	25
Plant 2 (MXN/Unit)	5	12	5.5	12.5	8
Plant 3 (INR/Unit)	60	40	65	39	32

Table 5.13 Unit inventory cost of PCs at each DC

DC	USA (USD/Unit)	Britain (GBP/Unit)	Canada (CAD/Unit)	Germany (EUR/Unit)	Japan (JPY/Unit)
Inventory cost	25	13.75	29.5	20.475	2700

Table 5.14 Unit outsourcing inventory cost of PCs for each DC

DC	USA (USD/Unit)	Britain (GBP/Unit)	Canada (CAD/Unit)	Germany (EUR/Unit)	Japan (JPY/Unit)
Inventory cost	30	16.75	35.5	25.475	3100

Table 5.15 Inventory capacity of PCs at each DC

DC	1	2	3	4	5
Capacity (Unit)	250	270	290	240	230

Table 5.16 Expected value of currency exchange rates in each sub-period

Sub-period / Country	1	2	3	4
Thailand	0.0240	0.0242	0.0243	0.0241
Mexico	0.0860	0.0865	0.0862	0.0863
India	0.0218	0.0217	0.0215	0.0217
USA	1	1	1	1
Britain	1.82	1.83	1.80	1.79
Canada	0.8456	0.8455	0.8454	0.8455
Germany	1.23	1.24	1.22	1.21
Japan	0.0091	0.0092	0.0093	0.0094

Table 5.17 Revenue tax rate in each country

Country	Thailand	Mexico	India	USA	Britain	Canada	Germany	Japan
tax	10%	20%	15%	35%	30%	13.7%	15%	20%

Table 5.18 Allowable intervals for transfer pricing

Plant	Thailand(THB/unit)	Mexico(MXN/unit)	India(INR/unit)
Transfer price range	[89000,93000]	[24000,26000]	[100000,104000]

Table 5.19 Market price of PCs at each demand market

Demand market	1 (USD/Unit)	2 (GBP/Unit)	3 (CAD/Unit)	4 (EUR/Unit)	5 (JPY/Unit)
Price	2200	1210	2620	1780	244000

Table 5.20 Mean of normal distribution for the stochastic demand in each sub-period at each demand market

Sub-period \ Demand market	1	2	3	4	5
1	200	210	220	190	180
2	210	190	200	190	200
3	190	180	165	170	190
4	200	210	220	190	188

Table 5.21 Scenario 1 of standard deviation of normal distribution for the stochastic demand in each sub-period at each demand market

Sub-period \ Demand market	1	2	3	4	5
1	40	42	40	39	43
2	30	29	28	33	31
3	52	51	50	53	51
4	21	22	23	20	19

We assume that the confidence level in defining the chance constraint for each DC is 80%, i.e., $\alpha_j = 0.2, j = 1, 2, \cdots, 5$, and that the inventory of raw materials and PCs in plants or DCs at the beginning of sub-period 1 is 0. Without loss of generality, all of the duties are assumed to be 0. In addition, it is set that $\tau^{(0)} = 1000000$, $\hat{\tau} = 100$, $\sigma = 0.9$, $\beta = 1.5$, $\varepsilon = 0.001$, $IteMax =$ 10 and $\mu_0 = 40$ for implementing the proposed solution algorithm for the example.

Figure 5.3 depicts changes of the objective function $\psi(X, \mu)$, shown in eqn. (5.35) , and the value of the penalty function shown in the second term of objective function $\psi(X, \mu)$ with the number of iterations for the penalty function method. According to Figure 5.3, it can be seen that the value of penalty function at the fifth iteration is equal to zero; this means that the solution is US$ 2.7×10^6 .

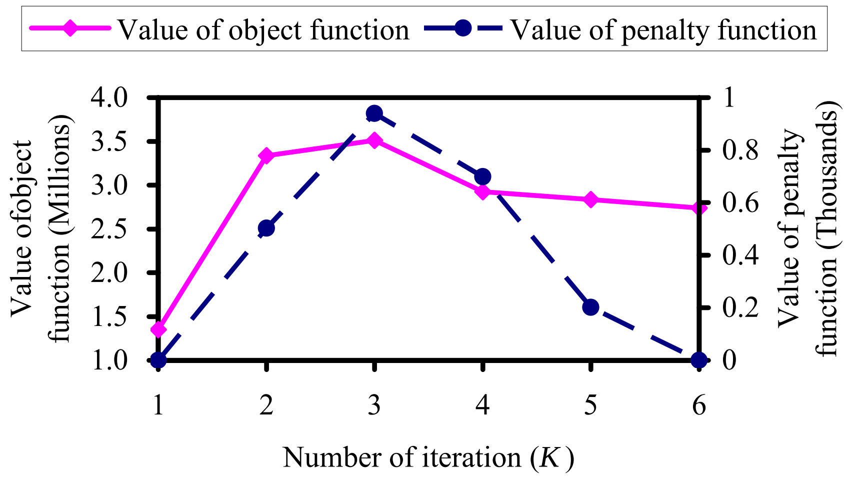


Figure 5.3 Convergent trend of the penalty function method embedded in the simulated annealing procedure

At each iteration, the penalty function method invokes the simulated annealing procedure for solving linearly constrained maximization problem (5.35) corresponding to a fixed penalty parameter μ. Figure 5.4 shows the performance of the simulated annealing procedure in solving the sub-problem (5.35) with $\mu = \mu_0\beta^5$. It clearly demonstrates the convergence trend of the simulated annealing procedure in solving the linearly constrained maximization problem (5.35).

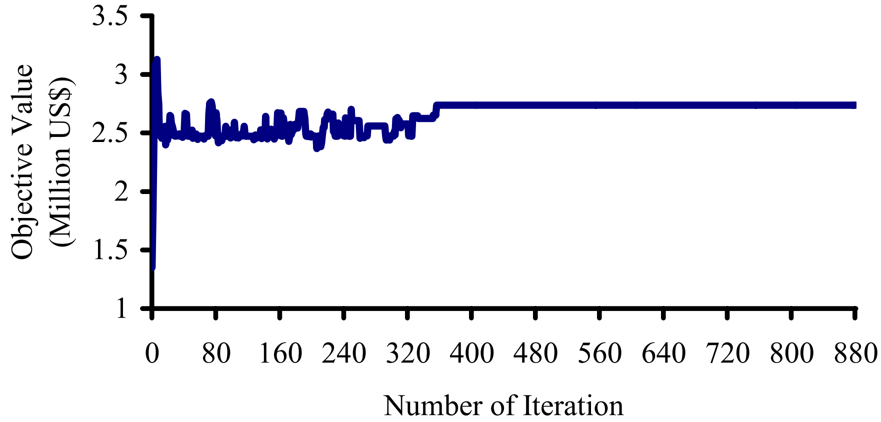


Figure 5.4 Convergence trend of the simulated annealing procedure in solving linearly constrained maximization problem (5.35) with parameter $\mu=\mu 0\beta^5$

Demand stochasticity has impact on the maximum expected value of the after-tax profit. It can be described by the standard deviation of the normal distribution shown in Table 5.21. In this presentation, four sets of standard deviation shown in Tables 5.21-5.24 were applied to the numerical example. Figure 5.5 shows that the maximum after-tax profit increases as the standard deviation increase.

Table 5.22 Scenario 2 of standard deviation of normal distribution for the stochastic demand in each sub-period at each demand market

Sub-period \ Demand market	1	2	3	4	5
1	45	47	45	44	48
2	35	34	33	38	36
3	57	56	55	58	56
4	26	27	28	25	24

Table 5.23 Scenario 3 of standard deviation of normal distribution for the stochastic demand in each sub-period at each demand market

Sub-period \ Demand market	1	2	3	4	5
1	50	52	50	49	53
2	40	39	38	43	41
3	62	61	60	63	61
4	31	32	33	30	29

Table 5.24 Scenario 4 of standard deviation of normal distribution for the stochastic demand in each sub-period at each demand market

Sub-period \ Demand market	1	2	3	4	5
1	55	57	55	54	58
2	45	44	43	48	46
3	67	66	65	68	66
4	36	37	38	35	34

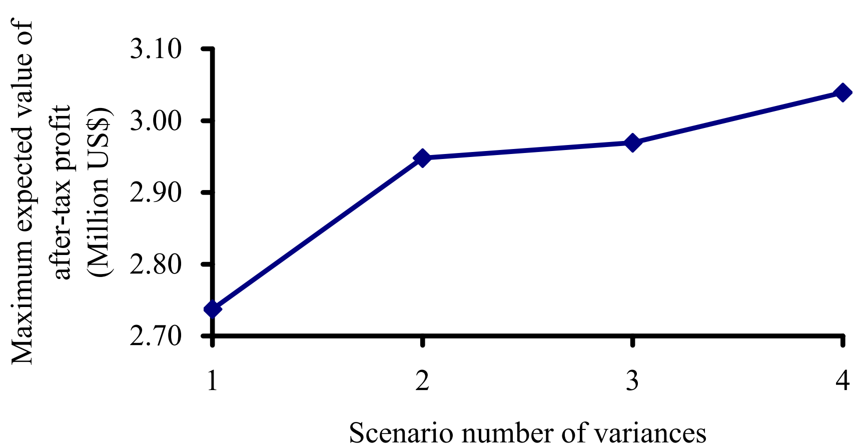


Figure 5.5 Changes of maximum expected value of after-tax profit with respect to four scenarios of standard deviation

Different confidence levels, each of which applies to all DCs are applied, to examine the impact of the chance constraints defined by equations (5.31) on the model. Figure 5.6 shows that the maximum expected after-tax profit value increases monotonically as the confidence level decreases.

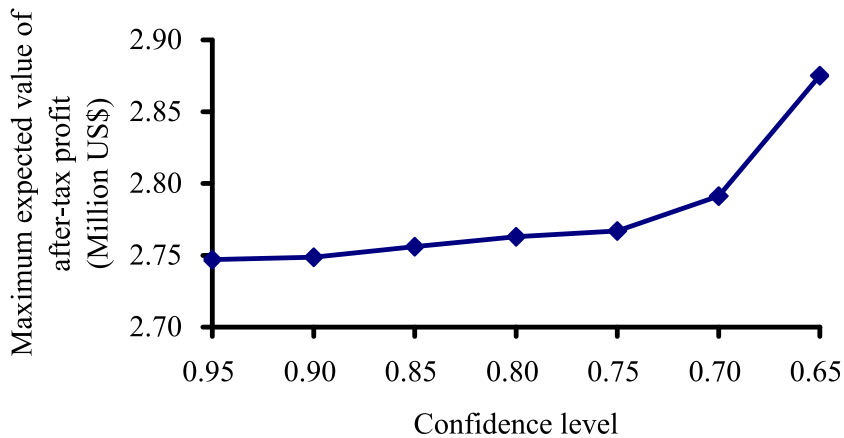


Figure 5.6 Changes of maximum expected value of after-tax profit with respect to different confidence levels

To numerically test the performance of the heuristic method proposed in section 4, 10 numerical examples are generated, whose values of parameters are in twenty percent difference with the values showed in Tables 5.1-5.21. Table 5.25 shows the time used to obtain the results of the 10 numerical examples.

Table 5.25 Computational time of the randomly generated numerical examples

No. of numerical example	Computational time (seconds)
1	15644
2	12369
3	16112
4	7850
5	13461
6	5322
7	10021
8	8098
9	6689
10	7891

5.6 Discussion and Summary

This chapter has developed a chance constrained programming model for a multination production-distribution planning for an MNC with consideration of transfer pricing and demand uncertainty. Maximization of the expected after-tax profit in the objective includes these decision variables: the order of raw materials for each plant, the inventory of both raw materials and products in each plant, the shipment of the products from each plant to each DC, the production of each plant and the transfer price charged by each plant, for all sub-periods. The proposed model can more or less help an MNC to plan his global supply chain.

To solve the model, this chapter has proposed a heuristic method that is a penalty function method embedded with the simulated annealing procedure that employs

Phrase I of the simplex method to perform the Metropolis Monte Carlo simulation. Numerical results not only demonstrate efficiency of the proposed heuristics method, but also analyze impacts of the stochastic demand and chance constraints on the expected profit.

CHAPTER 6 GAME-THEORETICAL MODEL FOR DECENTRALIZED GLOBAL SUPPLY CHAINS

6.1 Introduction

In the last chapter, the research is conducted on a global supply chain design problem with consideration of transfer pricing and demand uncertainty for an MNC. In this chapter, the research is moved to the competition of MNCs that are producing substitutable products and hence compete with each other with consideration of transfer pricing, allocation of transportation cost and gradual tax brackets for each MNC.

6.2 Problem Statement and Assumptions

Assume that a number of MNCs producing and selling substitutable products compete with each other worldwide via their respective two-echelon global supply chains comprising plants and DCs. These two-echelon global supply chains with market competition are referred to as the decentralized global supply chain. In each individual two-echelon global supply chain owned by an MNC, plants produce or assemble a product and DCs purchase the product from the plants and sell them to consumers. To maximize after-tax profit, each MNC involved in the decentralized global supply chains seek an optimal plan of production, distribution, pricing and transportation cost allocation, which consists of quantity of the product produced at a plant, price of the product quoted by a DC, transfer price, and the shipment of the product and transportation cost allocation between a plant and a DC. It is interested in characterizing and finding an equilibrium solution in terms of the production-distribution, pricing and transportation cost allocation plan for each MNC involved in the decentralized global supply chain by assuming that all the MNCs which are incentive to maximize their own after-tax profit compete each other without cooperation.

Given a two-echelon global supply chain owned by an MNC, it is assumed that each DC in the global supply chain buys the product from only one plant. Such an assumption is known as the single-sourcing strategy and has been used by Huang et al. (2005) and Romeijn et al. (2007). Since the currency exchange rate, import duty rate and income tax rate, transfer price, called the international economic parameters, vary over different countries, they should be thus taken into account by each MNC in making the optimal the production, distribution, pricing and transportation cost allocation plan for maximizing the after-tax profit. More interestingly, the MNC is able to coordinate transportation cost allocation between its plants and DCs to reduce tax paid because these plants and DCs belong to the same MNC.

Because corporate income tax rates of a country are usually comprised by several brackets, these tax brackets are numbered by consecutive integers starting from number 1, namely, $\{1, 2, ...,\}$, and assume that the larger the tax bracket number is, the bigger the income tax rate will be. It is further assumed that demand for the product at a DC owned by an MNC is a function of selling prices quoted by those DCs located in the same country or territory in the decentralized global supply chain.

The following is notation that will be used throughout this chapter, including indices, sets, parameters and decision variables.

Sets and indices

$\{1,2,...,C\}$	Set of MNCs involved in the decentralized global supply chains, indexed by c, where C is total number of the MNCs.
M	Set of countries where plants in the decentralized global supply chain are located, indexed by m.
N	Set of countries where DCs in the decentralized global supply chain are located, indexed by n.
P_m^c	Set of plants owned by MNC c and located in country $m \in M$, indexed by i.
W_{nm}^{ci}	Set of DCs owned by MNC c, located in country $n \in N$ and purchasing the product from plant $i \in P_m^c$, indexed by j.
$S_m = \{1,2,\cdots,\}$	Set of corporate income tax brackets of country $m \in M$, indexed by $\overline{s}$.
$S_n = \{1,2,\cdots,\}$	Set of corporate income tax brackets of country $n \in N$, indexed by $\hat{s}$.

International economic parameters

$DUTY_{mn}$	Import duty rate on the value of product shipped from country $m \in M$ to country $n \in N$.
E_m	Currency exchange rate of country $m \in M$ to US dollars [US dollars/monetary units of country m].

E_n	Currency exchange rate of country $n \in N$ to US dollars [US dollars/monetary units of country n].
TPR_m	Maximal transfer price perturbation range of product imposed by tax authority of country $m \in M$ [monetary units of country $m \in M$].
$TAX_m^{\bar{s}}$	Income tax rate for bracket $\bar{s}$ in country $m \in M$.
$TAX_n^{\hat{s}}$	Income tax rate for bracket $\hat{s}$ in country $n \in N$.
$U_m^{\bar{s}}$	Upper bound of tax bracket $\bar{s} \in S_m$ in country $m \in M$ [monetary units of country $m \in M$].
$U_n^{\hat{s}}$	Upper bound of tax bracket $\hat{s} \in S_n$ in country $n \in N$ [monetary units of country $n \in N$].

Costs, production capacity and demand function associated with MNC $c \in C$

CA_i^c	Production capacity of plant i owned by MNC c [unit of product].
PC_i^c	Unit production cost of the product produced in plant i of MNC c [monetary units of the country where plant i is located/ unit of product].
TC_{ij}^c	Unit transportation cost excluding import duty of product shipped from plant i of MNC c to DC j of MNC c [monetary units of the country where plant i is located /unit of product].
$D_j^c\left(z_j^c, \mathbf{z}_n^{-c}\right)$	Demand function for product at DC j located in country

$n \in N$, owned by MNC c, where z_j^c is selling price of the product quoted by DC j located in country $n \in N$, and $\mathbf{z}_n^{-c}$ is a row vector of all the selling prices of the product quoted by those DCs located in country n but owned by the other MNCs, namely,

$\mathbf{z}_n^{-c} = \left\{ z_j^k \middle| j \in W_{nm}^{ki}, i \in P_m^k, k = 1, 2, \cdots, c-1, c+1, ..., C \right\}$.

Decision variables associated with MNC $c \in C$

x_{ij}^c Amount of product, produced in plant i of MNC c and supplied to DC j of MNC c [amount of unit of product].

y_{ij}^c Transfer price charged by plant i of MNC c for product supplied to DC j of MNC c [US dollars/unit of product].

z_j^c Selling price of product quoted by DC j of MNC c [monetary units of the country where DC j is located /unit of product].

α_{ij}^c Fraction of transportation cost allocated to plant i of MNC c for transporting product from plant i of MNC c to DC j of MNC c.

In relation to the incorporate income tax rates, without loss of generality, it is assumed that

$$TAX_m^0 = 0; U_m^0 = 0; TAX_m^{\bar{s}-1} \le TAX_m^{\bar{s}} ; U_m^{\bar{s}-1} \le U_m^{\bar{s}}, \bar{s} \in S_m, m \in M \quad (6.1)$$

$$TAX_n^0 = 0; U_n^0 = 0; TAX_n^{\hat{s}-1} \le TAX_n^{\hat{s}} ; U_n^{\hat{s}-1} \le U_n^{\hat{s}}, \hat{s} \in S_n, n \in M \quad (6.2)$$

Eqns. (6.1)-(6.2) reflect the stepwise corporate income rate. For any DC j owned by

MNC c, it is assumed that demand function for the product has the partial linear expression:

$$D_j^c\left(z_j^c, \mathbf{z}_n^{-c}\right) = a_j^c - b_j^c z_j^c + f_j^c\left(\mathbf{z}_n^{-c}\right) \tag{6.3}$$

where a_j^c and b_j^c are two nonnegative parameters, and $f_j^c\left(\mathbf{z}_n^{-c}\right)$ is assumed to be a continuous differentiable function with respect to vector $\mathbf{z}_n^{-c}$. It should be pointed out that the price-sensitive linear demand function has been postulated by many game theoretical applications in domestic supply chain management (Corbett and Karmarker, 2001; Leng and Parlar, 2005). Our partial linear demand function defined by eqn. (6.3) is obviously more generic than those linear demand functions used in the literature.

6.3 Two Maximization Models to Characterize Behavior of an Individual MNC in Maximization of his After-profit

Let $\boldsymbol{\mu}^c$ be a row vector of all the decision variables defining a feasible plan of production, distribution, pricing and transportation cost allocation for MNC $c \in \{1, 2, \cdots, C\}$, namely,

$$\boldsymbol{\mu}^c = \left(x_{ij}^c, y_{ij}^c, z_j^c, \alpha_{ij}^c, i \in P_m^c, j \in W_{nm}^{ci}, m \in M, n \in N\right) \tag{6.4}$$

Given a row vector $\boldsymbol{\mu}^c$, revenues of MNC c in terms of US dollars gained from plant i and DC j can be expressed by two functions of decision variable $\boldsymbol{\mu}^c$, respectively:

$$\Psi_i^c\left(\boldsymbol{\mu}^c\right) = \sum_{n \in N} \sum_{j \in W_{nm}^{ci}} \left[\left(x_{ij}^c \times y_{ij}^c\right) - E_m \times \left(PC_i^c \times x_{ij}^c + \alpha_{ij}^c \times TC_{ij}^c \times x_{ij}^c\right)\right], i \in P_m^c, m \in M \tag{6.5}$$

$$\begin{aligned} \Psi_j^c\left(\boldsymbol{\mu}^c\right) = {} & \left(E_n \times z_j^c \times x_{ij}^c\right) - \left[\left(1 + DUTY_{mn}\right) \times x_{ij}^c \times y_{ij}^c\right] \\ & - \left[E_m \times \left(1 - \alpha_{ij}^c\right) \times TC_{ij}^c \times x_{ij}^c\right], i \in P_m^c, j \in W_{nm}^{ci}, n \in N, m \in M \end{aligned} \tag{6.6}$$

Eqn. (6.5) indicates that revenue generated by a plant of MNC c amounts to the income from selling the product to all the DCs less the sum of the production cost and the allocated transportation cost. Eqn. (6.6) implies that revenue yielded by a DC of MNC c is equal to the income from selling the product to consumers minus expense in purchasing the product from a plant and the relevant transportation cost allocated to the DC.

Identifying the optimal plan of production, distribution, pricing and transportation cost allocation to maximize the after-profit for MNC c can be mathematically formulated by the nonlinearly constrained maximization model with decision variables $\boldsymbol{\mu}^c$, dummy variables $\boldsymbol{\pi}^c$ and parameter $\mathbf{z}^{-c}$:

$$\begin{aligned}\max_{\boldsymbol{\mu}^c,\boldsymbol{\pi}^c} F_c\left(\boldsymbol{\mu}^c,\boldsymbol{\pi}^c,\mathbf{z}^{-c}\right) = & \sum_{m\in M}\sum_{i\in P_m^c}\left[\Psi_i^c\left(\boldsymbol{\mu}^c\right) - E_m \times \sum_{\bar{s}\in S_m}\left(TAX_m^{\bar{s}} \times \pi_i^{c\bar{s}}\right)\right] \\ & + \sum_{n\in N}\sum_{j\in W_{nm}^{ci}}\left[\Psi_j^c\left(\boldsymbol{\mu}^c\right) - E_n \times \sum_{\hat{s}\in S_n}\left(TAX_n^{\hat{s}} \times \pi_j^{c\hat{s}}\right)\right]\end{aligned} \tag{6.7}$$

subject to

$$\Psi_i^c\left(\boldsymbol{\mu}^c\right) \le E_m \times \sum_{\bar{s}\in S_m} \pi_i^{c\bar{s}}, i\in P_m^c, m\in M \tag{6.8}$$

$$\Psi_j^c\left(\boldsymbol{\mu}^c\right) \le E_n \times \sum_{\hat{s}\in S_n} \pi_j^{c\hat{s}}, i\in P_m^c, j\in W_{nm}^{ci}, m\in M, n\in N \tag{6.9}$$

$$\pi_i^{c\bar{s}} \le U_m^{\bar{s}} - U_m^{\bar{s}-1}, i\in P_m^c, m\in M, \bar{s}\in S_m \tag{6.10}$$

$$\pi_j^{c\hat{s}} \le U_n^{\hat{s}} - U_n^{\hat{s}-1}, j\in W_{nm}^{ci}, i\in P_m^c, m\in M, n\in N, \hat{s}\in S_n \tag{6.11}$$

$$\left(E_n \times z_j^c\right) - TPR_m \le y_{ij}^c \le \left(E_n \times z_j^c\right) + TPR_m, j\in W_{nm}^{ci}, i\in P_m^c, m\in M, n\in N \tag{6.12}$$

$$\sum_{n\in N}\sum_{j\in W_{nm}^{ci}} x_{ij}^c \le CA_i^c, i\in P_m^c, m\in M \tag{6.13}$$

$$x_{ij}^c \le a_j^c - b_j^c z_j^c + f_j^c\left(\mathbf{z}_n^{-c}\right), j\in W_{nm}^{ci}, i\in P_m^c, m\in M, n\in N \tag{6.14}$$

$$0 \le \alpha_{ij}^c \le 1, i\in P_m^c, j\in W_{nm}^{ci}, m\in M, n\in N \tag{6.15}$$

$$x_{ij}^c, y_{ij}^c, z_j^c \geq 0, i \in P_m^c, j \in W_{nm}^{ci}, m \in M, n \in N \tag{6.16}$$

$$\pi_i^{c\bar{s}} \geq 0, i \in P_m, m \in M, \bar{s} \in S_m \tag{6.17}$$

$$\pi_j^{c\hat{s}} \geq 0, j \in W_{nm}^{ci}, i \in P_m^c, m \in M, n \in N, \hat{s} \in S_n \tag{6.18}$$

where the row vector $\mathbf{z}^{-c} = \left(\mathbf{z}_n^{-c}, n \in N\right)$, and the row vector of all the dummy variables:

$$\boldsymbol{\pi}^c = \left(\pi_i^{c\bar{s}}, \pi_j^{c\hat{s}}, i \in P_m^c, j \in W_{nm}^{ci}, \bar{s} \in S_m, \hat{s} \in S_n, m \in M, n \in N\right) \tag{6.19}$$

The dummy variables in the bracket of right hand side of eqn. (6.19) play a role in converting the stepwise income tax rates, shown in eqns. (6.1)-(6.2), into two continuous differentiable parts defining the total after-tax profit of MNC c, shown in the right hand side of eqn. (6.7). Eqn. (6.7) is the objective function that is to maximize the after-tax profit of MNC c. Eqns. (6.8) and (6.9) partition the incomes of plant i and DC j into two components $\pi_i^{c\hat{s}}$ and $\pi_j^{c\hat{s}}$ corresponding to the gradual income tax brackets of countries m and n, respectively. Eqns. (6.10) and (6.11) imposes upper bounds on dummy decision variables $\pi_i^{c\bar{s}}$ and $\pi_j^{c\hat{s}}$. It can be seen that maximization of the objective function, together with constraints eqns. (6.8)-(6.10), guarantee the income tax paid following the gradual tax rates when a plant or a DC makes a positive income. Eqn. (6.12) imposes the bounds for transfer prices according to tax authorities' regulation. This is because according to OECD international guidelines based on arm's length principle, transfer prices are restricted in a certain range around the real market prices of a product. Eqn. (6.13) is the production capacity constraint for each plant. Eqn. (6.14) implies that the amount of the product purchased by DC j does not exceed demand that DC j faces. Eqn. (6.15) is a straightforward constraint for the transportation cost allocation ratio between a plant and a DC. Eqns. (6.15)-(6.18) show nonnegativity of all the decision variables including the dummy variables.

Given any fixed value of parameter $\mathbf{z}^{-c}$, it can be seen that model (6.7)-(6.18) is a nonlinear and non-concave maximization problem with respect to variables $\boldsymbol{\mu}^c$ and $\boldsymbol{\pi}^c$ because of these three terms, $y_{ij}^c x_{ij}^c$, $\alpha_{ij}^c x_{ij}^c$ and $z_j^c x_{ij}^c$, involved in revenue functions $\Psi_i^c\left(\boldsymbol{\mu}^c\right)$ and $\Psi_j^c\left(\boldsymbol{\mu}^c\right)$. Fortunately, a simple but useful mathematical transformation technique can be employed for the above non-concave maximization model to yield a concave maximization formulation. To do so, three groups of new variables are defined as follows:

$$\eta_{ij}^c = x_{ij}^c \times y_{ij}^c, i \in P_m^c, j \in W_{nm}^{ci}, m \in M, n \in N \tag{6.20}$$

$$v_{ij}^c = \alpha_{ij}^c \times x_{ij}^c, i \in P_m^c, j \in W_{nm}^{ci}, m \in M, n \in N \tag{6.21}$$

$$\gamma_{ij}^c = z_j^c \times x_{ij}^c, i \in P_m^c, j \in W_{nm}^{ci}, m \in M, n \in N \tag{6.22}$$

Let $\boldsymbol{\lambda}^c$ be a row vector of all the new variables and the distribution variables $\left\{x_{ij}^c\right\}$, namely,

$$\boldsymbol{\lambda}^c = \left(x_{ij}^c, \eta_{ij}^c, v_{ij}^c, \gamma_{ij}^c, i \in P_m^c, j \in W_{nm}^{ci}, m \in M, n \in N\right) \tag{6.23}$$

In terms of vector $\boldsymbol{\lambda}^c$, two revenue functions shown in eqns. (6.5)-(6.6) can be rewritten by

$$\Psi_i^c\left(\boldsymbol{\lambda}^c\right) = \sum_{n \in N} \sum_{j \in W_{nm}^{ci}} \left[\eta_{ij}^c - E_m \times \left(PC_i^c \times x_{ij}^c + TC_{ij}^c \times v_{ij}^c\right)\right], i \in P_m^c, m \in M \tag{6.24}$$

$$\begin{aligned}\Psi_j^c\left(\boldsymbol{\lambda}^c\right) = &\left(E_n \times \gamma_{ij}^c\right) - \left[\left(1 + DUTY_{mn}\right) \times \eta_{ij}^c\right] \\ &- \left[E_m \times TC_{ij}^c \times x_{ij}^c - E_m \times TC_{ij}^c \times v_{ij}^c\right], j \in W_{nm}^{ci}, i \in P_m^c, n \in N, m \in M\end{aligned} \tag{6.25}$$

According to Eqns. (6.24)-(6.25), it can be seen that functions $\Psi_{Pi}^c\left(\boldsymbol{\lambda}^c\right)$ and $\Psi_{DCj}^c\left(\boldsymbol{\lambda}^c\right)$ both are linear functions of vector $\boldsymbol{\lambda}^c$. In terms of variable $\boldsymbol{\lambda}^c$, dummy variable $\boldsymbol{\pi}^{cs}$, and parameters $\mathbf{x}^{-c}$ and $\boldsymbol{\gamma}^{-c}$, model (6.7)-(6.15) can be reformulated as follows:

$$\begin{aligned}\max F_c\left(\boldsymbol{\lambda}^c,\boldsymbol{\pi}^{cs},\mathbf{x}^{-c},\boldsymbol{\gamma}^{-c}\right) &= \sum_{m\in M}\sum_{i\in P_m^c}\left(\Psi_i^c\left(\boldsymbol{\lambda}^c\right)-E_m\times\sum_{\bar{s}\in S_m}TAX_m^{\bar{s}}\times\pi_i^{c\bar{s}}\right)\\ &+\sum_{n\in N}\sum_{j\in W_{nm}^{ci}}\left(\Psi_j^c\left(\boldsymbol{\lambda}^c\right)-E_n\times\sum_{\hat{s}\in S_n}TAX_n^{\hat{s}}\times\pi_j^{c\hat{s}}\right)\end{aligned} \tag{6.26}$$

subject to

$$\Psi_i^c\left(\boldsymbol{\lambda}^c\right)\le E_m\times\sum_{\bar{s}\in S_m}\pi_i^{c\bar{s}}, i\in P_m^c, m\in M \tag{6.27}$$

$$\Psi_j^c\left(\boldsymbol{\lambda}^c\right)\le E_n\times\sum_{\hat{s}\in S_n}\pi_j^{c\hat{s}}, j\in W_{nm}^{ci}, i\in P_m^c, m\in M, n\in N \tag{6.28}$$

$$\pi_i^{c\bar{s}}\le U_m^{\bar{s}}-U_m^{\bar{s}-1}, i\in P_m^c, m\in M, \bar{s}\in S_m \tag{6.29}$$

$$\pi_j^{c\hat{s}}\le U_n^{\hat{s}}-U_n^{\hat{s}-1}, j\in W_{nm}^{ci}, i\in P_m^c, m\in M, n\in N, \hat{s}\in S_n \tag{6.30}$$

$$\left(E_n\times\gamma_j^c\right)-\left(TPR_m\times x_{ij}^c\right)\le\eta_{ij}^c\le\left(E_n\times\gamma_j^c\right)+\left(TPR_m\times x_{ij}^c\right), j\in W_{nm}^{ci}, i\in P_m^c, m\in M, n\in N \tag{6.31}$$

$$\sum_{n\in N}\sum_{j\in W_{nm}^{ci}}x_{ij}^c\le CA_i^c, i\in P_m^c, m\in M \tag{6.32}$$

$$\left(x_{ij}^c\right)^2-a_j^c x_{ij}^c+b_j^c\gamma_{ij}^c-x_{ij}^c f_j^c\left(\boldsymbol{\gamma}_n^{-c},\mathbf{x}_n^{-c}\right)\le 0, j\in W_{nm}^{ci}, i\in P_m^c, m\in M, n\in N \tag{6.33}$$

$$0\le v_{ij}^c\le x_{ij}^c, j\in W_{nm}^{ci}, i\in P_m^c, m\in M, n\in N \tag{6.34}$$

$$x_{ij}^c,\eta_{ij}^c,\gamma_{ij}^c\ge 0, i\in P_m^c, j\in W_{nm}^{ci}, m\in M, n\in N \tag{6.35}$$

$$\pi_i^{c\bar{s}}\ge 0, i\in P_m, m\in M, \bar{s}\in S_m \tag{6.36}$$

$$\pi_j^{c\hat{s}}\ge 0, j\in W_{nm}^{ci}, i\in P_m^c, m\in M, n\in N, \hat{s}\in S_n \tag{6.37}$$

where the four row vectors associated with the other MNCs are:

$$\boldsymbol{\gamma}_n^{-c}=\left(x_{ij}^k z_j^k, j\in W_{nm}^{ki}, i\in P_m^k, m\in M, k=1,2,\cdots,c-1,c+1,\cdots,C\right) \tag{6.38}$$

$$\mathbf{x}_n^{-c}=\left(x_{ij}^k, j\in W_{nm}^{ki}, i\in P_m^k, m\in M, k=1,2,\cdots c-1,c+1,\cdots,C\right) \tag{6.39}$$

$$\boldsymbol{\gamma}^{-c}=\left(\boldsymbol{\gamma}_n^{-c}, n\in N\right) \tag{6.40}$$

$$\mathbf{x}^{-c}=\left(\mathbf{x}_n^{-c}, n\in N\right) \tag{6.41}$$

It can be seen that constraints (6.31), (6.33) and (6.34) are derived by multiplying x_{ij}^{c} for both sides of constraints (6.12), (6.14) and (6.15), respectively, due to nonnegativity of multiplier x_{ij}^{c}. As the left hand side of eqn. (6.33) is a convex function with respect to variable $\boldsymbol{\lambda}^{c}$, model (6.26)-(6.37) is hence a concave maximization problem with respect to variables $\boldsymbol{\lambda}^{c}$ and $\boldsymbol{\pi}^{cs}$, which can be effectively solved by several solution algorithms in nonlinear program. In addition, eqns. (6.20)-(6.22) implies the following proposition.

Proposition 6.1. Non-concave maximization model (6.7)-(6.18) and concave maximization model (6.26)-(6.37) are equivalent in the optimal solution and objective function value.

6.4 Generalized Nash Game Model

As the decentralized global supply chain consists of C MNCs producing substitutable products, these MNCs hence compete with each other without any cooperation in order to maximize their respective after-tax profit. This non-cooperative competition problem can be formulated by a generalized Nash game model. In terms of terminology used in game theory, each MNC is also regarded as a player whose behavior is described by either model (6.7)-(6.18) or model (6.26)-(6.37). Here model (6.26)-(6.37) is preferred due to its favorable concavity with efficient optimization solvers. As a result, the payoff function and strategy set of MNC (or player) $c \in \{1, 2, \cdots, C\}$ are the objective function shown in eqn. (6.26) and the feasible solution set defined by constraints (6.27)-(6.37), respectively.

To emphasize a strategy of a specified MNC $c \in \{1,2,\cdots,C\}$, let us define three row vectors:

$$\boldsymbol{\xi}^c = \left(\boldsymbol{\lambda}^c, \boldsymbol{\pi}^c\right), c = 1,2,\cdots,C \tag{6.42}$$

$$\boldsymbol{\xi}^{-c} = \left(\boldsymbol{\xi}^1, \boldsymbol{\xi}^2, \cdots, \boldsymbol{\xi}^{c-1}, \boldsymbol{\xi}^{c+1}, \cdots, \boldsymbol{\xi}^C\right), c = 1,2,\cdots,C \tag{6.43}$$

$$\boldsymbol{\xi} = \left(\boldsymbol{\xi}^c, \boldsymbol{\xi}^{-c}\right) \tag{6.44}$$

Vector $\boldsymbol{\xi}^c$ denotes a strategy (or a feasible solution) of player c (or MNC c); vector $\boldsymbol{\xi}^{-c}$ subsumes all the other MNCs' strategies except MNC c; vector $\boldsymbol{\xi}$ is a joint strategy of all the MNC. Given a fixed vector $\boldsymbol{\xi}^{-c}$, the rivals' strategies of MNC c, let $\Omega_c\left(\boldsymbol{\xi}^{-c}\right)$ be the set of all strategies of MNC c, namely,

$$\Omega\left(\boldsymbol{\xi}^{-c}\right) = \left\{\boldsymbol{\xi}^c \middle| \boldsymbol{\xi}^c \text{ satisfies constraints (6.27)-(6.37) } \right\}, c = 1,2,...,C \tag{6.45}$$

$\Omega\left(\boldsymbol{\xi}^{-c}\right)$ is the feasible solution set of the concave maximization model (6.26)-(6.37) associated with parameter $\boldsymbol{\xi}^{-c}$. Therefore, $\Omega\left(\boldsymbol{\xi}^{-c}\right)$ is a compact and convex set in the space $\Re^{\left|\boldsymbol{\xi}^c\right|}$, where $\left|\boldsymbol{\xi}^c\right|$ denotes dimension of vector $\boldsymbol{\xi}^c$. Let Ω be the set of all joint strategies for all the MNCs, namely,

$$\Omega = \left\{\boldsymbol{\xi} = \left(\boldsymbol{\xi}^c, \boldsymbol{\xi}^{-c}\right) \middle| \boldsymbol{\xi} \text{ satisfies constraints (6.27)-(6.37) for each MNC } c = 1,2,\cdots,C \right\} \tag{6.46}$$

The above set Ω is no longer convex in the space $\Re^{|\boldsymbol{\xi}|}$, where $|\boldsymbol{\xi}|$ denotes dimension of vector $\boldsymbol{\xi}$, because the left hand side of constraint (6.33) is a non-convex function in variable $\boldsymbol{\xi}$.

Given a joint strategy $\boldsymbol{\xi}$, let $\Omega(\boldsymbol{\xi})$ be Cartesian product of the corresponding strategy set shown in eqn. (6.45) of each MNC, namely,

$$\Omega(\boldsymbol{\xi}) = \Omega_1\left(\boldsymbol{\xi}^{-1}\right) \times \cdots \times \Omega_1\left(\boldsymbol{\xi}^{-C}\right) \tag{6.47}$$

Assuming that each MNC $c \in \{1,2,...,C\}$ follows passive perception, the game-theoretical model to characterize the equilibrium solution of the decentralized global supply chain can be formulated as follows:

Find a vector $\xi^* = \left(\xi^{*,1}, \xi^{*,2}, \cdots, \xi^{*,c}, \xi^{*,c+1}, \cdots, \xi^{*,C}\right) \in \Omega\left(\xi^*\right)$ such that

$$F_c\left(\xi^{*,c}, \xi^{*,-c}\right) \geq F_c\left(\xi^c, \xi^{*,-c}\right), \forall \xi^c \in \Omega\left(\xi^{*,-c}\right) \text{ for each player } c \in \{1,2,...,C\} \quad (6.48)$$

where payoff function $F_c(\cdot,\cdot)$ is the objective function shown in eqn. (6.26).

Eqn. (6.48) reflects an equilibrium situation that no MNC can increase its after-profit by unilaterally changing its own strategy compared to the joint strategy ξ^*. According to taxonomy of games (Harker, 1991), model (6.48) is nominated as the generalized Nash game, and any of its solution is called the generalized Nash equilibrium solution. With regarding to a specific MNC c, its strategy set $\Omega\left(\xi^{-c}\right)$ is a non-empty, closed and convex set for any given ξ^{-c}. In addition, payoff function $F_c\left(\xi^c, \xi^{-c}\right)$ is continuous in variable ξ and concave in variable ξ^c. Theorem 12.3 of Aubin (1998) guarantees the existence of a solution to the generalized Nash game model (6.48), namely:

Proposition 6.2. The generalized Nash game model (6.48) possesses at least one solution.

It should be pointed out that the solution to the generalized Nash game model (6.48) may not be unique. Uniqueness of the generalized Nash equilibrium solution needs very strong conditions on the payoff function and joint strategy set (Aubin, 1998).

6.5 Two Heuristic Methods

Compare to a normal Nash game, it is more challenging to develop an efficient and convergent solution method for solving a generalized Nash game model. This is because the strategy set of each player depends on the other players' strategies; see strategy set $\Omega\left(\xi^{-c}\right)$ of player c shown in eqn. (6.48). Up to now, operations research scholars have contributed two sorts of approaches for solving a generalized Nash game model: quasi-variational inequality and optimization formulation. Since a generalized Nash game model can be reformulated as a quasi-variational inequality (Harker, 1991), Chan and Pang (1982) and Pang and Fukushima (2005) proposed a projection method and a penalty function method for solving the induced quasi-variational inequality, respectively. These two solution methods are conceptual with computational difficulty in implementation even for small problems. More recently, Heusinger and Kanzow (2006) and Fukushima (2006) transformed a generalized Nash game model into an optimization formulation by making use of Nikaido-Isoda type function (Nikaido and Isoda, 1955), which allows us to employ any solution method solving optimization problems to find a generalized Nash equilibrium solution. Notice that Nikaido-Isoda function based solution methods have been employed to solve the normal Nash game models (Uryasev and Rubinstein, 1994; Krawczyk and Uryasev, 2000). However, all these solution methods need a fundamental assumption that the full Cartesian product of the strategy sets of a generalized Nash game model is convex; otherwise these methods are inapplicable. Unfortunately, the full Cartesian product of the strategy sets for the generalized Nash game model (6.48), namely, set Ω defined by eqn. (6.46), is a non-convex set. In other words, it is necessary to seek heuristic methods to find an equilibrium solution of model (6.48).

Given a fixed vector ξ^{-c}, strategies of the other MNCs, the optimal strategy of MNC c corresponding to ξ^{-c} can be obtained easily by solving the parameterized concave maximization model (6.26)-(6.37). Such a property enables us to seek an iterative solution method that needs to solve the concave maximization model (6.26)-(6.37) at each iteration. Inspired by the solution methods solving the normal Nash game model of Basar (1987), two heuristic methods are come out as follows.

Gauss-Seidel Iterative Method

Step 0: (Initiation) Choose an initial joint strategy $\xi^{(0)} = \left(\xi^{1^{(0)}},...,\xi^{C^{(0)}}\right) \in \Omega$, and let the number of iterations $k := 0$ and the number of players examined $c := 0$

Step 1: (Solve the concave maximization for MNC c) Let $c := c+1$, and find optimal solution $\xi^{c^{(k+1)}}$ that solves the concave maximization model (6.26)-(6.37) associated with given parameter $\left(\xi^{1^{(k)}},...,\xi^{c-1^{(k)}},\xi^{c+1^{(k)}},...,\xi^{C^{(k)}}\right)$.

Step 2: (Check the number of MNCs that have been examined) If $c < C$, go to Step 1. Otherwise, go to step 3.

Step 3: (Check a stop criterion) If $\max\limits_{c=1,2,...,C}\left|F_c\left(\xi^{c^{(k+1)}},\xi^{-c^{(k+1)}}\right) - F_c\left(\xi^{c^{(k)}},\xi^{-c^{(k)}}\right)\right| \le \varepsilon$, then stop, where ε is a predetermined stop tolerance; otherwise, set $k := k+1$, $c := 0$ and go to step 1.

In the above Gauss-Seidel iterative method, Steps 1-2 simulates the decision procedure of a MNC that acts on the other MNCs' strategies made in the last iteration. It can be easily checked that $\xi^{(k+1)}$ is a generalized Nash equilibrium solution if

$$\max_{c=1,2,\ldots,C}\left|F_c\left(\xi^{c^{(k+1)}},\xi^{-c^{(k+1)}}\right)-F_c\left(\xi^{c^{(k)}},\xi^{-c^{(k)}}\right)\right|=0 \tag{6.49}$$

In other words, the stop criterion of Step 3 is applicable. As an alternative, Steps 1-2 can be replaced with another computational process that assumes that the MNCs take their turns sequentially and act on the latest updated information obtained from the other MNCs, which leads to the second heuristic method:

Cournot Iterative Method

Step 0: (Initiation) Choose an initial joint strategy $\xi^{(0)}=\left(\xi^{1^{(0)}},\ldots,\xi^{C^{(0)}}\right)\in\Omega$, and let the number of iterations $k:=0$ and the number of players examined $c:=0$

Step 1: (Solve a concave maximization model of player c) Let $c:=c+1$, and find optimal solution $\xi^{c^{(k+1)}}$ that solves the concave maximization model (6.26)-(6.37) associated with given parameter $\left(\xi^{1^{(k+1)}},\ldots,\xi^{c-1^{(k+1)}},\xi^{c+1^{(k)}},\ldots,\xi^{C^{(k)}}\right)$.

Step 2: (Check the number of MNCs that have been examined) If $c<C$, go to Step 1. Otherwise, go to step 3.

Step 3: (Check a stop criterion) If $\max_{c=1,2,\ldots,C}\left|F_c\left(\xi^{c^{(k+1)}},\xi^{-c^{(k+1)}}\right)-F_c\left(\xi^{c^{(k)}},\xi^{-c^{(k)}}\right)\right|\le\varepsilon$, then stop, where ε is a predetermined stop tolerance; otherwise, set $k:=k+1$, $c:=0$ and go to step 1.

Compared the two heuristic methods presented above, difference only lies in the parameters defining the concave maximization model of Step 1. The latter heuristic method uses parameter $\left(\xi^{1^{(k+1)}},\ldots,\xi^{c-1^{(k+1)}},\xi^{c+1^{(k)}},\ldots,\xi^{C^{(k)}}\right)$ that includes the latest updated decisions $\xi^{1^{(k+1)}},\ldots,\xi^{c-1^{(k+1)}}$ made by MNCs from number 1 to number $c-1$.

6.6 Numerical Examples

To evaluate the generalized Nash game model (6.48) and assess two heuristic methods presented above, several illustrative numerical examples are used of the decentralized global supply chain. Two heuristic methods - Gauss-Seidel and Cournot iterative methods - are coded by Version 7.0 of Matlab, and both are run on a Desktop computer with CPU of Intel P4 3.00GHZ and RAM of 512M. The relevant concave maximization model (6.26)-(6.37) in Step 1 of these two heuristic methods is solved by optimization solver “fmincon” of Matlab. Moreover, the stop tolerance $\varepsilon = 10^{-6}$ is applied for all the numerical examples.

6.6.1 An example with two MNCs

To evaluate impact of currency exchange rate, income tax bracket and transportation allocation on the generalized Nash equilibrium solution, a numerical example of the decentralized global supply chain with two MNCs producing substitutable products as shown in Figure 6.1 is developed.

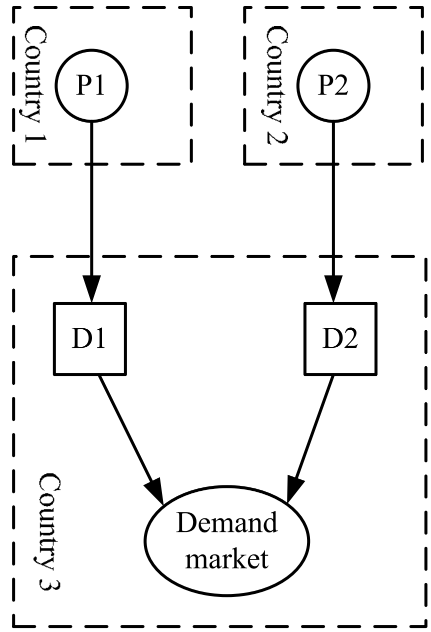


Figure 6.1 A decentralized global supply chain with two MNCs

It is assumed that the two-echelon global supply chain of MNC 1 comprises plant P1 and DC D1 and the two-echelon global supply chain of MNC 2 consists of plant P2 and DC D2. Plants P1 and P2 are located in Countries 1 and 2, respectively, while DCs D1 and D2 are both located in Country 3. It is further assumed that DCs D1 and D2 serve the same demand market.

Table 6.1 Currency exchange rate to US$ of each country

Country	1	2	3
Currency exchange rate	0.130	0.277	1.000

Table 6.2 Income tax brackets with different tax rates for each country

Country 1	Income tax bracket (10^5 monetary units of Country 1)	(0, 8]	(8,16]	(16,24]	$(24,+\infty)$
	Income tax rate	0.2	0.25	0.30	0.35
Country 2	Income tax bracket (10^5 monetary units of Country 2)	(0,3]	(3,6]	(6,9]	$(9,+\infty)$
	Income tax rate	0.30	0.35	0.40	0.45
Country 3	Income tax bracket (10^5 monetary units of Country 3)	(0,1]	(1,2]	(2,3]	$(3,+\infty)$
	Income tax rate	0.15	0.20	0.25	0.30

Table 6.3 Import duty rate ($DUTY_{mn}$) between two countries

From \ To	Country 3
Country 1	0.20
Country 2	0.30

Table 6.4 Unit production cost, unit transportation cost and production capacity for each plant

Plant \ Parameter	Unit production cost (PC_i^c)	Unit transportation cost (TC_{ij}^c)	Production capacity (CP_i^c)
Plant 1	1000	30	2000
Plant 2	520	15	2000

Table 6.5 Maximum transfer price perturbation range imposed by tax authority of each country

Perturbation range	Country 1	Country 2
TPR_m (US$)	100	150

For each country, currency exchange rate and 4 income tax brackets with the relevant tax rates are tabulated in Tables 6.1 and 6.2, respectively. Table 6.3 gives the import duty rates from Country 1 to Country 3 and from Country 2 to Country 3. Table 6.4 shows the unit production cost, the unit transportation cost and the production capacity for each plant; Table 6.5 lists the maximum transfer price perturbation range imposed by tax authority of each country. It is further assumed that the demand functions of two DCs have the following expressions:

$$D_1\left(z_1^1, z_2^2\right) = 1000 - 2z_1^1 + 1.5z_2^2 \tag{6.50}$$

$$D_2\left(z_1^1, z_2^2\right) = 1200 - 2z_2^2 + 1.5z_1^1 \tag{6.51}$$

6.6.1.1 Impact of currency exchange rate

Figures 6.2 and 6.3 respectively depict changes of the after-tax profits and market prices of the product at the generalized Nash equilibrium solution, obtained by either of the two heuristic methods, with respect to different currency exchange rates of Country 1.

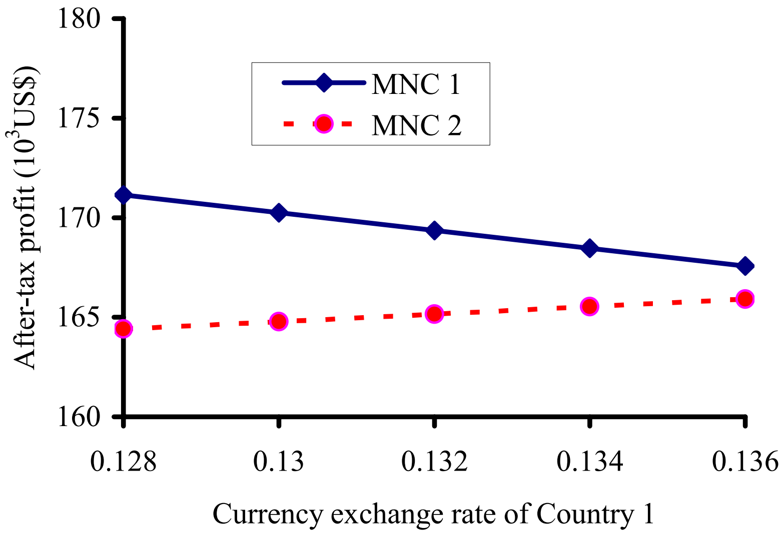


Figure 6.2 Impact of currency exchange rate of Country 1 on the after-tax profit

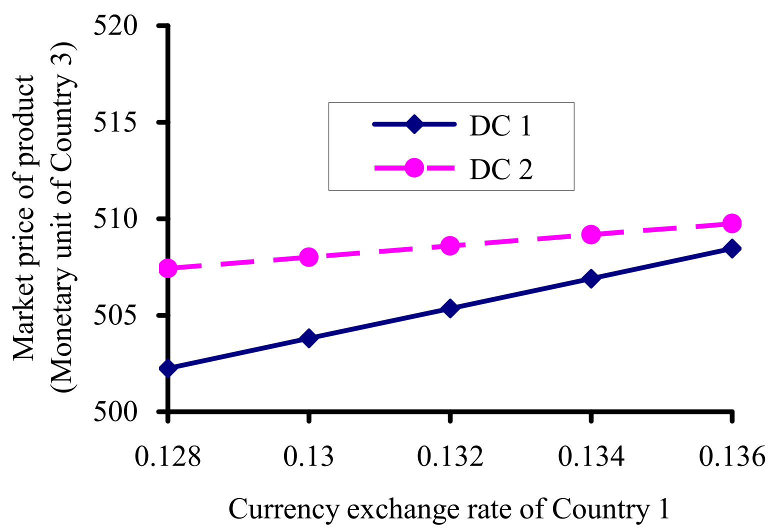


Figure 6.3 Impact of currency exchange rate of Country 1 on the market price of product

From Figure 6.2, it can be seen that the after-tax profit of MNC 1 declines with increase in currency exchange rate of Country 1 while the after-tax profit of MNC 2 increases. As shown in Figure 6.3, both market prices of the product quoted by MNCs 1 and 2 increases with currency exchange rate of Country 1.

Results shown in Figures 6.2 and 6.3 can be collaborated as follows. With increasing currency change rate of Country 1 where plant of MNC 1 is located, the production cost in raw material purchase and transportation cost of MNC 1 will be also be driven upwards. Hence in order to cover the increased cost, DC 1 of MNC 1 may tend to increase the market price of its product. This action will lead to a decrease in the demand faced by DC1 due to the market competition with MNC 2. As a result, the resulting after-tax profit of MNC 1 decreases because the increase in price is unable to cover the lost incurred from the decrease in demand. On the other hand, demand faced by DC 2 will increase due to increase of the market price of the product from DC 1. Accordingly, upon seeing this potential, MNC 2 will tend to increase its market price to obtain more after-tax profit. Therefore, the after-tax profit of MNC 2 increases.

6.6.1.2 Impact of income tax rates

To investigate impact of income tax rate on the transfer price and transportation allocation, three sets of income tax rates of Country 3, shown in Table 6.6, are used. Figure 6.4 illustrates change of transfer prices charged by Plants P1 and P2 with respect to the three sets of tax rates. By looking at Figure 6.4, it can be observed that both transfer prices increase with increase of tax rates of Country 3. This is because when Country 3 (where two DCs are located) increases its income tax rates, each MNC will increase its transfer prices to shift their profits from DCs to plants in order to reduce the income tax paid in Country 3.

Table 6.6 Three sets of income tax brackets with different income tax rates for Country 3

Tax brackets (10^5 monetary units of Country 3) / Set number of income tax rates	(0,1]	(1,2]	(2,3]	(3,+∞)
Set 1	0.20	0.25	0.3	0.35
Set 2	0.30	0.35	0.4	0.45
Set 3	0.40	0.45	0.5	0.55

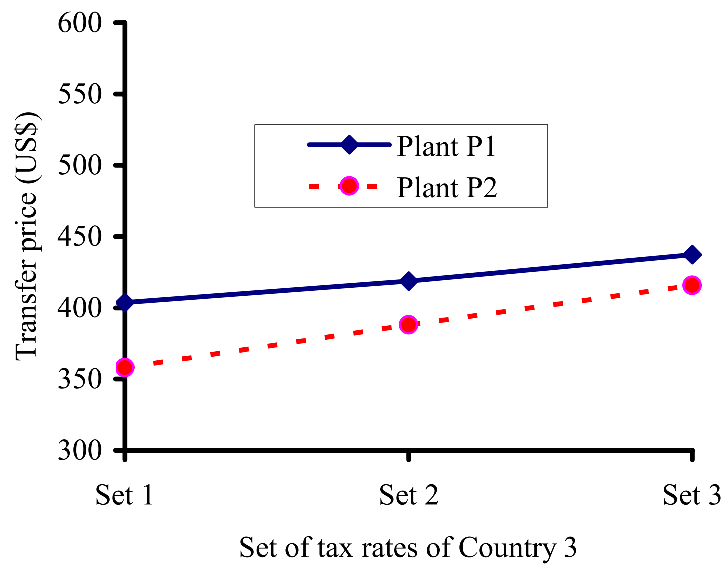


Figure 6.4 Impact of tax rates of Country 3 on transfer prices

Table 6.7 shows the transportation cost allocation ratios assigned to each plant with respect to the three set of income tax rates of Country 3 listed in Table 6.6. According to this table, MNC 1 will allocate all transportation cost to its DC D1 when the income tax rate of Country 3 increases to Set 3. This is because such an allocation can reduce the income tax paid in Country 3. In other words, MNC 1 can gain more after-tax profit by coordinating transportation allocation between its plants and DCs.

Table 6.7 Transportation cost allocation ratios $\left(\alpha_{ij}\right)$ for the two plants

Plants / Set number of income tax rates	P1	P2
Set 1	1.0	1.0
Set 2	1.0	1.0
Set 3	0	1.0

Tables 6.8 and 6.9 give three sets of income tax rates of Countries 1 and 2, respectively. Figures 6.5 and 6.6 plot transfer prices charged by Plants P1 and P2 with respect two scenarios of income tax rates shown in Tables 6.8 and 6.9, respectively. According to these two figures, it can be clearly seen that transfer price charged by a plant decrease with increase of the income tax rates of the country where the plant located in. These two figures also shows that change of tax rate of one country will not only affect the transfer price of plants located in the country, but also affect transfer prices of plants located in other countries due to the competition. In addition, the impact on the transfer prices of plants located in other countries is not as much as

the impact on the transfer prices of plant located in the country where the tax rates change.

Table 6.8 Three sets of income tax brackets with different income tax rates for Country 1

Set number of income tax rates \ Tax brackets (10^5 monetary units of Country 1)	(0, 8]	(8,16]	(16,24]	$(24, +\infty)$
Set 1	0.30	0.35	0.40	0.45
Set 2	0.40	0.45	0.50	0.55
Set 3	0.50	0.55	0.60	0.65

Table 6.9 Three sets of income tax brackets with different income tax rates for Country 2

Set number of income tax rates \ Tax brackets(10^5 monetary units of country 2)	(0,3]	(3,6]	(6,9]	$(9, +\infty)$
Set 1	0.15	0.2	0.25	0.3
Set 2	0.25	0.3	0.35	0.4
Set 3	0.35	0.4	0.45	0.5

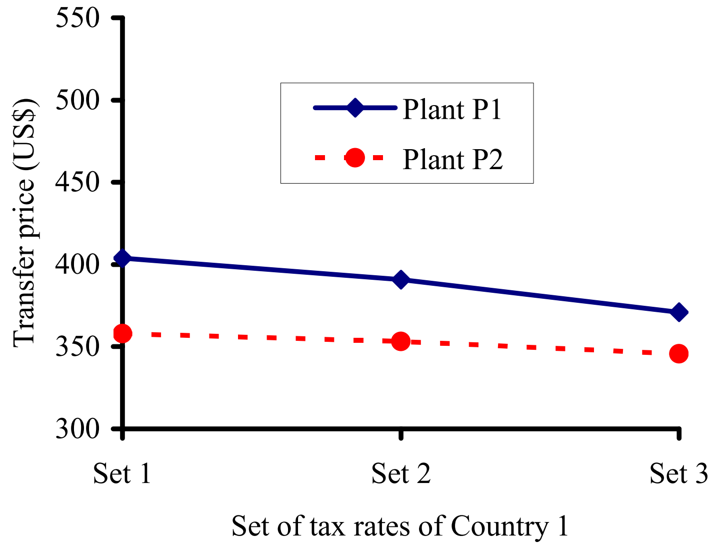


Figure 6.5 Impact of tax rates of Country 1 on transfer prices

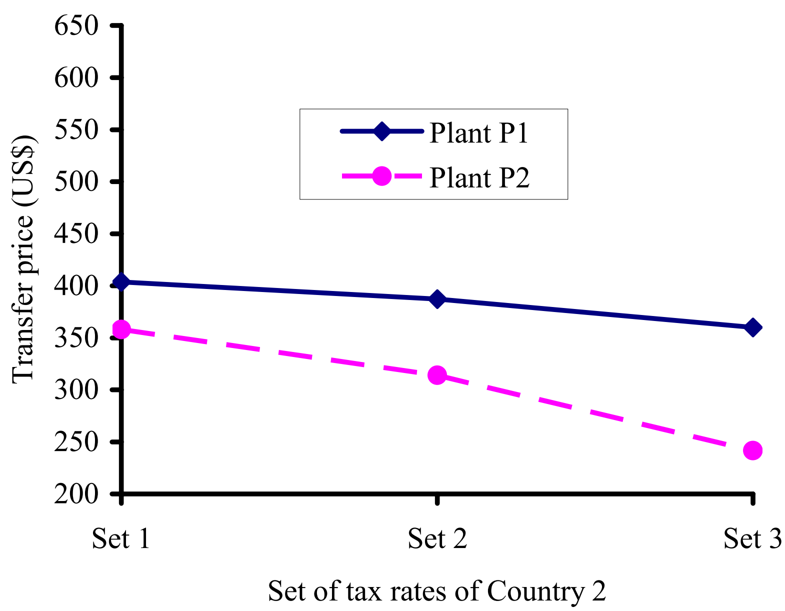


Figure 6.6 Impact of tax rates of Country 2 on transfer prices

6.6.2 Performance of two heuristic methods

Two scenarios of the decentralized global supply chain, shown in Table 6.10 was designed to numerically evaluate the performance of two heuristic methods. Scenario A consists of the decentralized global supply chain with a network structure shown in Figure 6.1. It is further assumed that each Country imposes 4 income tax brackets with different income tax rates and that the demand function takes the linear form like the one shown in eqn. (6.50) or (6.51).

Table 6.10 Two scenarios of the decentralized global supply chain

Configuration \ Scenario	A	B
Number of MNCs	2	5
Number of plants of each MNC	1	2
Number of DCs of each MNC	1	5
Number of decision variables for each MNC	14	48

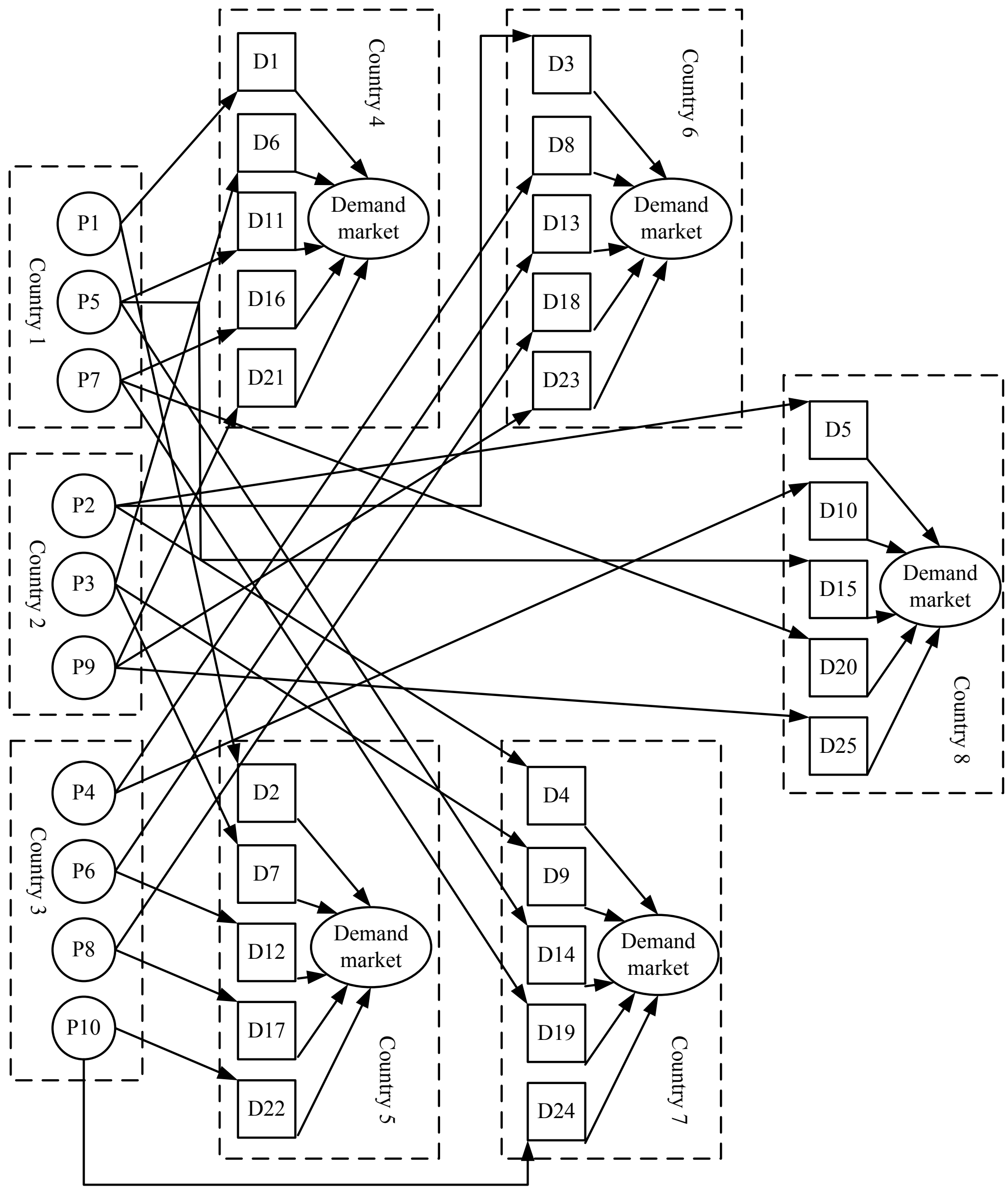


Figure 6.7 The decentralized supply chain of Scenario B

Regarding Scenario B, Figure 6.7 depicts the network structure of the decentralized global supply chain, which involves 5 MNCs and each MNC owns 2 plants and 5 DCs. More specifically, the two-echelon global supply chain of MNC 1 consists of plants P1, P2 and DCs D1 to D5; the two-echelon global supply chain of MNC 2 possesses plants P3, P4 and DCs D6 to D10; the two-echelon supply chain of

MNC 3 comprises plants P5, P6 and DCs D11 to D15; the two-echelon global supply chain of MNC 4 has plants P7, P8 and DCs D16 to D20; the two-echelon global supply chain of MNC 5 includes plants P9, P10 and DCs D21 to D25. Countries where these plants and DCs are located are shown in Figure 6.7. In addition, it is assumed that each country imposes 4 income tax brackets with different income tax rates. The demand function of a DC takes the following expression:

$$D_j^c\left(z_j^c, \mathbf{z}_n^{-c}\right) = a_1 - a_2 \times z_j^c + a_3 \times \sum_{c' \neq c, c'=1}^{C} \sum_{j' \in W_{nm'}^{c'i'}, i' \in P_m'} z_{j'}^{c'} \\ , j \in W_{nm}^{ci}, i \in P_m^c, n \in N, m \in M \tag{6.52}$$

where a_1 is real number in the interval $[2000, 3000]$; a_2 is a real number in the interval $[3.0, 4.0]$; a_3 is a real number in the interval $[0.8, 1.1]$.

For each scenario, values of those parameters involved in the concave maximization model (6.26)-(6.37) within the corresponding interval to yield 10 examples are generated. Fortunately, both of the two heuristic methods can find a solution for each of these twenty numerical examples. CPU time and the number of iterations used by these two heuristic methods are summarized in Tables 6.11 and 6.12, respectively. These two tables clearly indicate that the CPU time used by either of the two heuristic methods depends on the problem size. Finally, it is emphasized again that these two solution methods are heuristic rather than the convergent methods.

Table 6.11 CPU time and the number of iterations used by the Gauss-Seidel iterative method

Example No. (Scenario A)	CPU Time(s)	Number of Iterations	Example No. (Scenario B)	CPU Time(s)	Number of Iteration
1	25	14	1	1093	7
2	30	13	2	1138	7
3	33	15	3	1291	8
4	18	11	4	4925	23
5	21	12	5	4503	23
6	28	13	6	2627	17
7	43	26	7	2883	17
8	19	16	8	2635	14
9	38	22	9	2010	10
10	37	21	10	2331	12

Table 6.12 CPU time and the number of iterations used by the Cournot Iterative Method

Example No. (Scenario A)	CPU Time(s)	Number of Iterations	Example No. (Scenario B)	CPU Time(s)	Number of Iteration
1	22	13	1	1533	10
2	20	11	2	1631	12
3	35	17	3	1288	9
4	39	20	4	1649	13
5	30	15	5	4435	26
6	28	14	6	3012	19
7	53	28	7	2833	21
8	33	18	8	2001	15
9	42	21	9	1732	13
10	44	21	10	1305	9

6.7 Discussion and Summary

In this chapter a generalized Nash game model is proposed to investigate the equilibrium solution in terms of production, distribution, pricing and transportation cost allocation plan for each MNC involved in the decentralized global supply chain. After demonstrating existence of the generalized Nash equilibrium solution, two heuristic methods are employed to find an equilibrium solution, which need to solve a concave maximization model that reflects the behavior of an individual MNC in maximizing his after-tax profit. An example with two MNCs is applied to show the impact of international features on the equilibrium solution of the game-theoretical model proposed in this chapter. Moreover, twenty numerical examples randomly generated from two scenarios were carried out to numerically access the performance of two heuristic methods. As long as the demand function and other data is got, the proposed model in this chapter can help an MNC to anticipate his after-tax profit at the equilibrium state.

CHAPTER 7 CONCLUSIONS, RESEARCH CONTRIBUTION AND RECOMMENDATIONS FOR FUGURE RESEARCH

7.1 Conclusions

In this thesis, mathematical models and algorithms is developed for some issues in supply chain design and planning.

The unconstrained minimization formulations for the SCNE models were successfully derived. They enable us to employ the quasi-Newton algorithm to obtain the SCNE solution. Compared to the modified projection method, quasi-Newton algorithm does not need to predetermine the step size since a line search at each iteration obtains an optimal step size. Furthermore, the numerical results showed that for most cases the computational time used by quasi-Newton algorithm is less than that used by the modified projection method.

To reflect the production capacity constraints, the SCNE model with production capacity constraints was proposed. The numerical results showed that the outputs of equilibrium solutions with and without production capacity constraints are different, namely, the production capacity does affect the equilibrium solution of a decentralized supply chain.

Subsequently, an MPEC model was developed for a competitive facility location problem by using the SCNE model with production capacity constraints as the equilibrium constraints to describe the economic equilibrium state of a supply chain. The results showed that the maximal profit and total expenditure of establishing facilities increase as the budget for establishing facilities increase, and they will stop increasing as the budget is large enough. This interesting result means that an entering firm that wants to choose the location for opening its manufacturing facilities, opening more manufacturing facilities may not lead to more profit. This is because the competition exists among the newly opened and existing manufacturing facilities.

Regarding global supply chain planning, a chance-constrained programming model was proposed for multiperiod production-distribution planning for an MNC with consideration of transfer pricing and demand uncertainty. Sensitivity analysis of the confidence level and the standard deviation of the demand were studied. The numerical results showed that the expected value of the after-tax profit increases as the confidence level of inventories at DCs decreases or the standard deviation of the demand increases.

Finally, a generalized Nash game model was proposed for MNCs that produce substitutable products and compete with each other by considering transfer pricing, allocation of transportation cost and the gradual tax brackets for each MNC. Although 20 numerical examples demonstrated the convergence of Gauss-Seidel Iterative Method and Cournot Iterative Method, the convergence of the solution algorithms is yet to be proven. Actually, the solution algorithm for solving generalized Nash game model is an open question. To the best of our knowledge, up to date there is no efficient algorithm whose convergence has been proven for the generalized Nash game whose joint strategy set is nonconvex. It is a potential research topic in supply chain design and planning.

7.2 Research Contribution

The major contributions of this thesis are summarized as follows:

1. A comprehensive literature review of SCNE models, competitive facility location problems and global supply chain planning is provided.
2. An alternative formulation and solution method are investigated for SCNE models (Nagurney et al., 2002; Dong, et al., 2004). More specifically, the VI formulations for the SCNE models is transformed to unconstrained minimization problems and hence the quasi-Newton algorithm can be applied to solve it. The solution method, quasi-Newton algorithm, overcomes the limitation that it is impossible to find a universal step size while implementing the modified projection method for solving the SCNE models.
3. The SCNE model with production capacity constraints is developed. The modified projection method is unusable to solve this model because of the existence of capacity constraints. Therefore, the logarithmic-quadratic proximal prediction-correction (LQP P-C) method is investigated. A numerical example is applied to show the impact of production capacities of manufacturers on the equilibrium state of a supply chain.
4. A novel and interesting research issue regarding the competitive facility location on decentralized supply chains is explored. More specifically, an MPEC model is proposed for a competitive facility location by applying the SCNE model with production capacity constraints to describe the economic equilibrium state of the decentralized supply chain. A hybrid Genetic

algorithm (GA) incorporated with LQP P-C method is developed for solving this model. This is the first time that an equilibrium model that can describe the economic equilibrium of a decentralized supply chain comprising manufacturers, retailers and demand markets is applied to in a competitive facility location problem.

5. A chance-constrained programming model is built for the optimal production-distribution planning for an MNC with consideration of transfer pricing and demand uncertainty. A penalty function method incorporated with simulated annealing procedure is then presented for solving this model. This model would capture the fluctuation of currency exchange rates over a taxation period and the demand uncertainty, which have not been considered together with transfer pricing for an MNC so far.
6. A generalized Nash game model is developed to analyze the competition of MNCs that produce substitutable products. This is the first game-theoretical model for analyzing the competition of MNCs, taking into account transfer pricing, allocation of transportation cost and graduate tax brackets. Two heuristic methods are investigated and numerically analyzed.

7.3 Recommendation for Future Research

This thesis has only investigated a few interesting issues in supply chain design and planning. There are still many opportunities for further study on it. Following are several recommendations for the future study:

1. The competitive facility location problem studied in Chapter 3 assumes that there is one entering firm. As an extension, the game theory can be employed to examine multiple entering firms.
2. The currency exchange rate is one of the most important international features in global supply chain. In practice, currency exchange rates of different countries are dependent. With taking into consideration of dependent currency exchange rates, a novel research topic on global supply chain planning will be emerged. To our best knowledge, it has not been examined up to now.
3. The game-theoretical model proposed in Chapter 6 is actually a generalized Nash equilibrium problem. Up to date, algorithms for solving generalized Nash equilibrium problem are restricted in some kinds of special cases. Future research can be conducted on evolving efficient algorithms to solve the generalized Nash equilibrium problems.
4. The game-theoretical model proposed in Chapter 6 assumes that the market demand is deterministic. Hence, a model to study the Nash game of multiple MNCs that are facing random demand can be developed in the future.

Overall, the research in this thesis is a significant step of further understanding of mathematical models and algorithms for supply chain design and planning. It may have a potential in future research with regards to its importance and application in the field of academic.

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APPENDIX: RESEARCH ACCOMPLISHMENTS

Journal Papers:

(1) Meng, Q., Huang, Y. K. and Cheu, R.L. (2007) A note on supply chain network equilibrium models. *Transportation Research* **41E**, 60-71.

(2) Meng, Q., Huang, Y. K. and Cheu, R. L. Competitive facility location on decentralized supply chains. Accepted by European Journal of Operational Research in 2008.

(3) Meng, Q., Huang, Y. K. and Wang, X. B. Multiperiod global supply chain design with demand uncertainty. Submitted to Transportation Research Part E in Feb. 2007.

(4) Huang, Y. K. and Meng, Q. A game-theoretical model for decentralized global supply chains. Submitted to Transportation Research Part E in June, 2007.

Conference Papers:

(1) Meng Q., Huang, Y. K. and Cheu, R. L. (2004) A decentralized supply chain network design problem with equilibrium constraints. *Proceedings of The 9th Conference of Hong Kong Society for Transportation Studies*, 67-76.

(2) Meng, Q., Huang, Y. K. and Cheu, R. L. (2005) Unconstrained minimization formulations for the supply chain network equilibrium models. *Proceedings of The First International Conference on Transportation Logistics*.

(3) Meng, Q., Khoo, H. L., Huang Y. K. and Cheu, R. L. (2006) An MPEC model for the optimal controlflow operation problem with user equilibrium constraints. In:

Proceedings of the 9th International Conference on Applications of Advanced Technology in Transportation.

(4) Meng, Q and Huang, Y. K. (2006) Modelling a multiperiod global supply chain network design problem with transfer pricing, In *Proceeding of The 36th International Conference on Computers & Industrial Engineering*, CD-ROM, Taipei.

(5) Meng, Q, Khoo, H. L., Huang, Y. K. and Cheu, R. L. (2006) An MPEC model for the optimal contraflow operation problem with user equilibrium constraints, In *Proceeding of the Ninth International Conference on Applications of Advanced Technology in Transportation*, Chicago: American Society of Civil Engineering.